The Cambridge Handbook of Healthcare: Productivity, Efficiency, Effectiveness. *Cambridge University Press*; 2024:612-656.

18

Dynamic Assignment of Patients to Primary and Secondary Inpatient Units: Is Patience a Virtue?

Soroush Saghafian, Derya Kilinc, and Stephen J. Traub

Various hospitals in the US and around the world suffer from the well-known problem of emergency department (ED) overcrowding, which prevents them from serving their patients in effective and efficient ways. An important contributor to this problem, which became even more dire after the COVID-19 pandemic, is prolonged boarding of patients who are admitted to inpatient units through the ED. Patients admitted through the ED constitute about 50% of all nonobstetrical hospital admissions in the US and may be boarded in the ED for long hours with the hope of finding an available bed in their primary inpatient unit. In this chapter, we shed light on effective ways of reducing ED boarding times by considering the trade-off between keeping patients in the ED and assigning them to a secondary inpatient unit. The former can increase the risk of adverse events and cause congestion in the ED (which in turn, prevents serving new ED patients in a timely manner), whereas the latter may adversely impact the quality of care post-ED service. Further complicating this calculus is the fact that a secondary inpatient unit for a currently boarded ED patient can be the primary unit for a future arriving patient; assignments, therefore, should be made in an orchestrated way. Developing a queueing-based Markov decision process, we demonstrate that patience in transferring patients is a virtue, but only up to a point. We also find that, contrary to the prevalent perception, idling inpatient beds in hospitals can be beneficial (under some circumstances). Because the optimal policy for dynamically assigning patients to their primary and secondary inpatient units is complex and hard to implement in hospitals, we develop a simple policy that we term penalty-adjusted largest expected workload cost (LEWC-p). Using simulation analyses calibrated with hospital data, we find that implementing this policy could significantly help hospitals to improve their patient safety by reducing boarding times while controlling the overflow of patients to secondary units. Using data analyses and various simulation experiments, we also help hospital administrators by generating insights into hospital conditions under which achievable improvements are significant.

"Ah, all things come to those who wait."

—*Violet Fane (1843–1905)*

18.1 INTRODUCTION

Over the last decade, hospital emergency department (ED) overcrowding has become a widely recognized problem in healthcare delivery in the US and around the world. In a report to Congress, the US Government Accountability Office highlighted this problem and emphasized that ED waiting times for the emergent patients exceed the recommended time window for 50% of visits (GAO, 2009). The situation became even more dire after the COVID-19 pandemic with EDs serving record number of patients.[1]

Overcrowding in the ED may have sever consequences, including higher complication rates and even increased mortality (Bernstein et al., 2009; CNN, 2008). As overcrowding increases, patients are subject to higher dissatisfaction, impaired access, higher rates of leaving without being seen (LWBS), and decreased economic performance (Hoot and Aronsky, 2008).

One important factor associated with ED overcrowding is the prolonged ED boarding of patients admitted to inpatient units (GAO, 2003). Such ED boarding occurs when an admitted patient cannot be moved out of the ED due to bed unavailability in a downstream hospital unit. Although this may cause congestion and may block ED resources from being assigned to newly arrived patients (see, e.g., Saghafian et al., 2012, and the references therein for the bed-block effect), boarding may be viewed in a positive light by some when it is done to ensure transfer to the most appropriate inpatient unit (rather than a secondary unit that has bed availability). This is due to a questionable yet prevalent belief that patience in transferring admitted patients is always a virtue. This belief deserves further scrutiny, especially because it is well understood that prolonged boarding times have several negative consequences for patients, including an increased risk of adverse events (ROAE).[2]

[1] The Emergency Medical Treatment and Labor Act (EMTALA) passed in 1986 mandates that US EDs serve all arriving patients, regardless of their insurance or financial status. Thus, unlike many other service systems, EDs cannot easily close their doors. Furthermore, mechanisms such as ambulance diversion is not legal in some states; hence, many EDs have to continue accepting patients, even when they are overcrowded.

[2] Patients boarded in the ED are sometimes kept on hallway beds, which raises additional concerns about whether they receive the care that is deemed necessary for them – the inpatient unit level of care.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

The assignment of hospital beds to patients is a challenging task due to several complexities, including limited capacity of hospital beds, time-dependencies of bed request arrivals, and unique treatment requirements of patients (Proudlove et al., 2007). These complexities force hospital administrators to incorporate various aspects of the operational status of their system (such as the current congestion level, time of the day, and discharge times in inpatient units) in their decision-making process. Nevertheless, from a medical standpoint, the ideal way of assigning a bed for a specific type of patient is directly related to the patient's medical diagnosis and treatment needs. However, to accommodate patient demands with the limited available hospital resources, hospital administrators may consider alternative assignment options. In particular, when there is no available bed in the ideal downstream unit (i.e., the patient's primary inpatient unit), the patient may be assigned to an alternative, secondary inpatient unit with an acceptable (if suboptimal) service capability and capacity. This practice of assigning patients to an alternative unit is known as "overflowing."

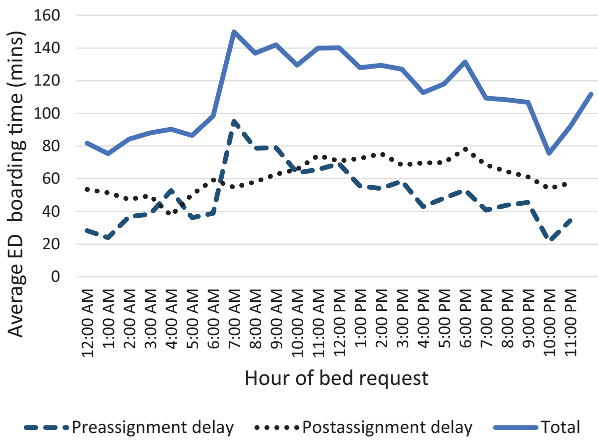
Overflowing is not a new concept in hospitals; however, in practice the overflow process is often controlled in a myopic manner without much attention to the needs of future patients. Instead, what is needed is a reasonable balance between the risk of keeping a patient in the ED (with the hope of a primary unit assignment) versus that of assigning the patient to a secondary unit that has current bed availability. A careful consideration of these trade-offs might have a significant impact on both patient safety and operational efficiency of hospitals, and thus, it can enable them to improve both the effectiveness and the efficiency of how they serve their patients. Our goal in this chapter is to discuss a systematic approach through which these objectives can be achieved.

To gain insights, we explore these issues in our partner hospital, Mayo Clinic Arizona (MCA). The data we have collected from MCA shows that the average ED boarding time (the average time between bed request and occupancy) at MCA is 111 minutes, with boarding times up to 150 minutes for some patients. Moreover, we observe from our data that about 30% of the patients admitted through the MCA ED are boarded for at least two hours. An average delay of 111 minutes is significant, especially when we consider that the average ED length of stay (LOS) for admitted patients in MCA is about five hours. This suggests that, on average, almost 37% of ED LOS is caused by boarding.[3] Furthermore, as is shown in Figure 18.1(a), we find that boarding duration is highly time-dependent. Therefore, even if the average waiting time is not extremely long, patients admitted through the ED experience different levels of delay based on the hour in which their inpatient bed is requested.

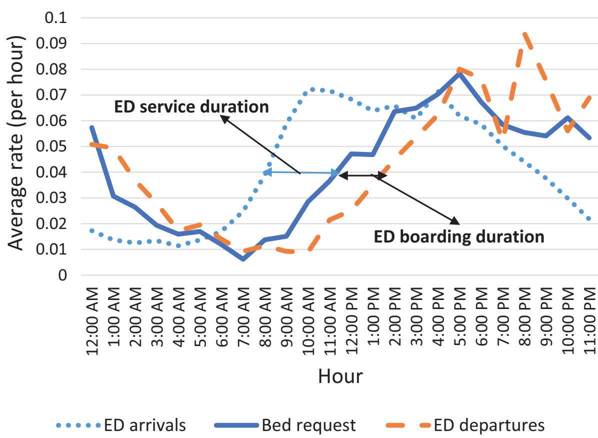
As we illustrate in Figure 18.1(a), boarding delays consists of two parts: preassignment and postassignment. Preassignment delay is the time between

[3] See also Carr et al. (2010), who report that 17% of the ED total LOS is caused by the ED boarding.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press



(a) Average boarding time



(b) Nonstationary arrival, bed request, and departure rates

FIGURE 18.1 ED boarding times based on collected data from our partner hospital

bed request and assignment of a suitable inpatient bed to the patient. Postassignment delay is the time between bed assignment and bed occupancy. Our analyses of MCA data reveal that postassignment delays are higher on average than preassignment delays (see Figure 18.1(a)). Additionally, as we show in Figure 18.1(b), there is a significant mismatch (i.e., time lag) between the hourly bed request pattern and the ED departure pattern (see, e.g., Shi et al., 2015; Powell et al., 2012, for related results reported for other hospitals). The time between ED arrivals and ED departures in Figure 18.1(b) is defined as ED LOS, and the time between bed requests and ED departures represents the ED boarding time. As can be seen from this figure, the ratio of ED boarding time to ED LOS can be as high as 48% for some patients.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

Effective assignment policies to primary and secondary inpatient units might significantly help hospitals such as MCA to improve their prolonged ED boarding times. To assist hospitals, we seek to answer the following questions:

- *Structure:* When should a patient be kept in the ED until a bed becomes available in their primary inpatient unit instead of being quickly assigned to a secondary unit with current bed availability?
- *Magnitude:* How much improvement can be achieved if a hospital adopts an effective policy for dynamically assigning ED admitted patients to their primary or secondary inpatient units?

To address these questions, we follow a two-step approach. As the first step, we start by analyzing the complex patient flow from the ED to inpatient wards through a simplified and stylized queuing model. This stylized model is tractable, and thus, we are able to analyze it analytically. This, in turn, allows us to generate clear insights into the structure question we raised here. As the second step, we then perform simulations calibrated with hospital data to (1) test the validity of our insights in a more realistic setting and (2) address our magnitude question.

For the first step, we make use of a Markov decision process (MDP) and study the flow process as a multiclass queueing system with "flexible" servers. In this setting, the servers are defined as the downstream inpatient unit beds that are flexible, in that they can serve different classes of patients. The literature on hospital-like multiclass queueing systems with flexible servers that can address the appropriateness of bed assignment decisions is not vast. We contribute to this literature by considering (1) a stochastic penalty cost that reflects the reduction in service quality when a patient is assigned to a secondary inpatient unit and (2) stochastic risk of adverse events that can occur due to prolonged ED boarding times. By analyzing our MDP setting, we find that the optimal assignment policy is a state-dependent threshold-type policy: Keeping patients in the ED for their primary inpatient unit to become available pays off, but only up to a certain threshold that depends on the number and status of outstanding ED bed requests. That is, *patience is a virtue, but only up to a point*.

Our findings and results regarding the structure of the optimal policy can help hospitals make better bed assignment decisions, particularly as we shed light on some guidelines that can strike a better balance between patient safety, quality of care, and operational efficiency. However, we note that the optimal policy generated by our model is complex to use in practice because it is highly dependent upon the system state (e.g., the number of patients of different types boarded in the ED and IW availabilities). Therefore, based on the properties of the optimal policy, we develop two heuristic policies that are simple to implement and effective. We test these heuristic policies by comparing their performance with the optimal policy and identify the policy that results in the smallest optimality gap. For the second step, we make use of a simulation model calibrated with data from our partner hospital and find that implementing our proposed assignment policy would reduce the average ED boarding time by 10 minutes per patient (a 9% improvement). Moreover, our simulation analysis suggests that our proposed policy would improve a combined measure of patient safety and quality of care metrics by 14% and would decrease the percentage of patients with more than two hours of boarding by 2%.

We also use our simulation framework to generate insights into hospital conditions under which such improvements can be most significant. Our results suggest that hospitals with higher congestion levels (e.g., busy teaching hospitals) would benefit more than other hospitals (e.g., less busy community hospitals) from utilizing our proposed policy as a way to strike a better balance between patient safety, quality of care, and operational efficiency. Our results also indicate that, under specific conditions on adverse event rates and the number of patients boarded in the ED, keeping an inpatient bed idle for potential future bed requests is beneficial. This practice of intentional bed idling is currently used in some inpatient units such as the intensive care unit (ICU). However, our results provide support for implementation across a wider range of inpatient units and reveal that bed idling should be used more broadly in hospitals. In fact, we observe from our analyses that the optimal policy strategically idles the available IW beds for future arriving patients in a variety of circumstances.

Our main contributions in this chapter are four-fold: (1) We generate insights into effective hospital bed assignment policies by developing a model that considers the trade-offs between risk of adverse events that may occur while a patient is boarded in ED and a potentially lower quality of care that might be provided if the patient is routed to a secondary unit. (2) We develop an easy-to-implement and yet effective policy for bed assignment in hospitals that considers multiple inpatient units, multiple patient types, time-dependent bed request arrivals, and dynamic ED and inpatient unit congestion levels. (3) By making use of some laboratory findings, and testing our proposed bed-assignment policy via a detailed simulation model calibrated with hospital data, we generate various insights for hospitals. For example, we find that our proposed policy is more effective in reducing ED boarding times for patients that are less sensitive to assignment to a secondary inpatient unit. Examples of such patients include those without an elevated serum troponin (Tn) level among chest pain (CP) patients or those with a B-type natriuretic peptide (BNP) less than 4,000 pg/ml among congestive heart failure (CHF) patients. (4) We also shed light on various hospital-dependent conditions under which our proposed policy is reasonably effective, thereby discussing the suitability of our proposed policy for implementation in a wide range of hospitals.

The rest of this chapter is organized as follows. Section 18.2 reviews the related studies on patient flow and dynamic assignment policies. Section 18.3 presents a model of patient flow and develops an MDP framework that captures

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

the trade-offs in the model. Section 18.4 identifies the structure of the optimal policy. In Section 18.5, we describe our proposed heuristic bed assignment policy and compare its performance with the optimal policy. In Section 18.6, we describe our detailed simulation model of patient flow and use it to perform various sensitivity analyses. Finally, we present our concluding remarks in Section 18.7. All proofs are provided in the online Appendix 18.C.

18.2 LITERATURE REVIEW

In this section, we briefly review studies that are related to our work. We divide such studies to two categories: (1) related studies on ED patient flow and (2) related studies on dynamic assignment and routing in queueing systems.

18.2.1 Related Studies on ED Patient Flow

Emergency department patient-flow studies can be found in both the medical and operations-research/management-science literature. In the medical literature, for example, Traub et al. (2016b) develop a novel algorithm for assigning arriving patients to physicians. Traub et al. (2015) provide evidence on the effectiveness of a new method of assessing ED patients up front and dispositioning them known as rapid medical assessment (RMA). Traub et al. (2016a) discuss the effectiveness of "physician in triage," a practice in which the ED patient flow is influenced by using physicians up front (i.e., in the triage stage). Traub et al. (2017) show that making use of holding orders can improve the ED patient flow by helping the transition of admitted ED patients to hospital beds. Hodgson et al. (2018) compare resource utilization of ED physicians with their admission practices, and Traub et al. (2018) shed light on the interphysician differences in improving ED patient flow. In the operations-research/management-science literature, Saghafian et al. (2012) show how a patient-flow design known as "streaming" can improve the performance of EDs. Saghafian et al. (2014) demonstrate that separating ED patients based on their medical complexity via a complexity-augmented triage algorithm, and redesigning the ED flow accordingly, can help EDs improve both their patient safety and operational efficiency. Saghafian et al. (2018) investigate the use of telemedical physicians in the ED, and Saghafian et al. (2022a) examine whether and how ED physicians might influence each other's performance. Finally, Saghafian et al. (2022b) study the characteristics of ED physicians that are highly effective and efficient and shed light on what such physicians do differently than their peers.

Studies on patient flow typically focus on patient flow either into the ED, within the ED, or out of the ED. An extensive review of operations-research/management-science contributions to these three elements can be found in Saghafian et al. (2015). Our work in this chapter focuses on patient flow out of the ED.[4] Research on this last part of flow includes studies on effective ways for improving the process for those who are admitted to the hospital through the ED as well as those discharged to go home. Our findings mainly contribute to the former, and hence, in what follows we only discuss the relevant studies within that literature.

Thompson et al. (2009) study a capacity utilization-based patient allocation problem. In their model, patients may be transferred between different units to minimize the total cost under a preemptive service policy assumption, where assignment to each unit is accompanied by a reward/cost. Similar to Thompson et al. (2009), we consider different levels of quality of care that can be provided in different inpatient units. However, unlike their study, we also model the risk of adverse events (ROAE) that can occur because of prolonged waiting in the ED. This allows us to provide a system-wide view that, in addition to operational efficiency, considers both patient safety and quality of care concerns. Another related study is Mandelbaum et al. (2012), which considers the fair routing of patients to inpatient units, where fair routing means targeting the same level of idleness among all servers. Unlike Mandelbaum et al. (2012), we consider patient routing as a mechanism to eliminate prolonged ED boarding times. Furthermore, the study of Mandelbaum et al. (2012) analyses a model with a single customer class, whereas we consider heterogeneous patient classes in order to gain insights into the questions we raised in the introduction. The effect of overflows on inpatient LOS is analyzed in Dong et al. (2018) and Song et al. (2019). Although these studies show that assigning patients to alternative units can result in higher LOS, we do not observe such an effect for the patients in our data set. Therefore, we build our model by considering service durations that depend on the patient's class but are independent of their final destination in the hospital.

Shi et al. (2015) focus on patient flow from ED to inpatient units and propose early discharge policies in inpatients units as a mechanism to reduce and flatten ED boarding times. Our study focuses on a similar patient flow from the ED to inpatient units; however, unlike the predetermined trigger times in Shi et al. (2015), we (1) optimize bed assignment decisions based on the number of boarded patients in the ED and (2) consider both patient safety and quality of care metrics. Furthermore, a policy of changing physician discharge routines that is described in Shi et al. (2015) might be hard to implement in many hospitals due to cultural issues such as difference in physicians' preferences. Our study offers guidelines on alternative ways of improving the patient flow. Another study that models overflow decisions is Dai and Shi (2019), where

[4] As discussed in Saghafian et al. (2015), elements of flow management into the ED include various aspects such as ambulance deployment and diversion as well as alternative roles of emergency medical services (e.g., in identifying nonurgent patients and redirecting them). Similarly, flow management within the ED include general triage interventions, strategic ED bed planning, staffing, and scheduling, among others.

approximate dynamic programming (ADP) model is used to obtain effective policies for routing patients to IWs.

Similar to our study, Griffin (2012) develops a patient-flow model to improve bed assignment by maximizing the suitability of patient assignments and minimizing ED boarding times. The author evaluates five dynamic bed assignment algorithms to aid decision-makers. Due to the large dimension of the state and action spaces, Griffin (2012) cannot identify the exact structure of the optimal assignment policy. In our study, we first gain insights into the structure of the optimal policy by using a stylized model of patient flow with two inpatient units and two patient types. We then make use of these insights to develop a heuristic policy. Using realistic simulations calibrated with hospital data, we next examine the performance of this heuristic policy in a realistic setting. This combination of analytical and simulation analyses allows us to fully address the questions we raised in the introduction. In addition, instead of assuming that all inpatient units can serve as a potential secondary unit for all patients (as is assumed in the majority of the previously mentioned studies), we use historical hospital data, laboratory findings, and physicians' opinion to determine specific secondary inpatient units for each patient type.

18.2.2 Related Studies on Dynamic Assignment and Routing in Queueing Systems

Gaining a deeper understanding of superior ways of assigning patients to providers (and routing them accordingly) through studying appropriate queueing systems can yield novel insights into improving ED operations (see, e.g., Saghafian et al., 2018, and the references therein). Our model in this chapter captures the system characteristics as a multiclass queueing system where the bed requests for ED admitted patients are considered arrivals and inpatient unit beds are considered servers. In multiclass queueing systems, the customers can be differentiated based on service rates, holding costs, arrival rates, or service requirements. Under an average holding cost objective, Cox and Smith (1991) demonstrate that the widely used $c\mu$ policy is optimal for both preemptive and nonpreemptive service protocols. The $c\mu$ policy is also shown to be the optimal policy in various more complex queueing networks (see, e.g., Kakalik and Little, 1971; Buyukkoc et al., 1985). A version of the $c\mu$ rule, generalized $c\mu$, is proved to be the optimal policy for different queueing structures under heavy traffic (see, e.g., Van Mieghem, 1995; Mandelbaum and Stolyar, 2004).

In Lin and Kumar (1984), the authors show that when two types of servers with different service speeds are available – a setting termed "slow server problem" – the optimal assignment policy is a threshold-type policy: Customers/jobs are assigned to the slow server whenever the queue length reaches a certain threshold. Our model resembles similar characteristics to the "slow server problem" because (1) patient service times in inpatient units are not identical and (2) there is some flexibility in assignments (for some patients).

However, instead of heterogeneous servers, we consider heterogeneous patient types with different service rates because it better matches the hospital patient flow we study. This differentiates our study from the previously mentioned studies in the literature because in such studies the resulted optimal policy typically depends on the difference between service rates of servers (see, e.g., Bell and Williams, 2001). However, our data analysis shows that service durations in primary and secondary units are not statistically different (among patients of a given type).

Dynamic assignment problems in queueing networks are extensively analyzed in the literature (see, e.g., Mandelbaum et al., 2012; Meyn, 2003). Armony and Bambos (2003) and Dai and Lin (2005) study dynamic assignment problems considering a throughput maximization objective. Andradóttir et al. (2007) and Saghafian et al. (2011) allow for server disruptions and repairs in systems with heterogeneous flexible servers, and De Véricourt and Zhou (2005) study a call center setting where agents are heterogeneous in terms of both service rate and quality of service (see also Zhan and Ward, 2013). Another related stream of literature that considers flexible servers is the skill-based routing literature, where the customers are routed to the servers that have the appropriate skill sets (similar to the routing of patients to primary versus secondary units in our study). However, unlike our work, the focus of those studies are mostly on settings where (1) servers have multiple skills (e.g., call center agents) and (2) staffing decisions are the primary concerns (see, e.g., Garnett and Mandelbaum, 2000; Wallace and Whitt, 2005). There are also various other studies on routing policies in multiserver, multiclass settings (see, e.g., Gurvich and Whitt, 2009; Armony and Ward, 2010; Gurvich and Perry, 2012). However, in these studies only costs related to waiting and losing customers are considered, whereas we focus on the trade-off between waiting and overflows. Finally, we note that the majority of the previously mentioned studies focus on heavy traffic settings. Unlike them, we seek to address the questions we raised in the introduction under practical hospital congestion levels. To this end, we do not impose any heavy traffic assumption and instead make use of actual hospital bed census data as the basis of our analytical and simulation analysis.

18.3 THE MODEL

A general representation of patient flow through the ED and hospital inpatient wards (IWs) is presented in Figure 18.2. A patient that arrives to the ED goes through the triage stage and is assigned an emergency severity index (ESI).[5] If there is an examination room available, the patient immediately starts the ED service; otherwise, they will have to wait in a designated ED waiting area. Once the ED treatment is done, the patient is either discharged home or is admitted to

[5] For more information regarding ESI and how it is used, see, e.g., Saghafian et al. (2014).

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

the hospital. For an admitted patient, if there is a bed available in their primary IW (or the secondary IW if applicable), the patient is transferred out of the ED; otherwise, they are kept in the ED until such a bed becomes available. For the goals of this study, we focus on the patient flow within the dashed area of Figure 18.2.[6]

To gain insights into the questions we raised in the introduction, we start by modeling the patient flow as a multiclass queueing system with IWs as flexible servers and analyze it by using an MDP. The patients in the system are classified based on their primary IW (i.e., where they can be best served from a medical standpoint). Ward-level placement is typically determined by a bed placement coordinator, sometimes in consultation with the ED or the admitting physician. Once a patient is moved to an IW, the IW bed is considered unavailable until the patient is done with the inpatient unit service, and hence, the service processes in IWs are typically nonpreemptive.

To gain some high-level insights, we start by considering each of the IWs as a single "super server," which represents the capacity of the IW as a whole. This pooling of beds within each IW allows us to keep track of availability of capacity in IWs in a computationally tractable way. However, to test the insights we gain from this simplifying assumption, we relax it in Section 18.6 and consider each IW bed as a server. Similarly, we start by considering the arrival process as a stationary Poisson process and assume IW service times are exponential. In Section 18.6, we also relax these simplifying assumptions by making use of empirical distributions (for both interarrival and service times) that we have estimated from our data.

Figure 18.3 illustrates the patient flow under consideration as a queueing system. Our discussions with medical providers revealed that, for the vast majority of patients, only one IW can be considered as a secondary IW.[7] Thus, as illustrated in Figure 18.3, the hospital network can conceptually be considered as an aggregated collection of primary and secondary IWs, where each patient type has only one primary IW and only one secondary IW.

To analyze the patient flow depicted in Figure 18.3, we let N_p and N_s denote the set of patient classes and servers (IWs), respectively. For $i \in N_p$, we denote by λ_i the arrival (i.e., bed request) rate of class i patients. We model the service process in IWs with class-dependent service rates μ_i where $i \in N_p$. We also let $X_i(t)$ denote the number of class i patients boarded in the ED at time t and define $\underline{X}(t) = (X_i(t) : i \in N_p)$ as the vector of the number of all such patients.

6 Thus, we do not consider measures related to events that occur outside this flow. For example, an important measures for EDs is the percentage of patients who leave without being seen (i.e., abandon the queue). But this occurs almost always from the waiting room of EDs (i.e., before the ED service starts), which is outside the dashed area in Figure 18.2.

7 We also note that some patients can only be served in their primary unit (e.g., ICU patients). We still consider a primary-secondary pair for such patients but disallow for service in the secondary IW by considering a high penalty cost for care delivery in the secondary IW.

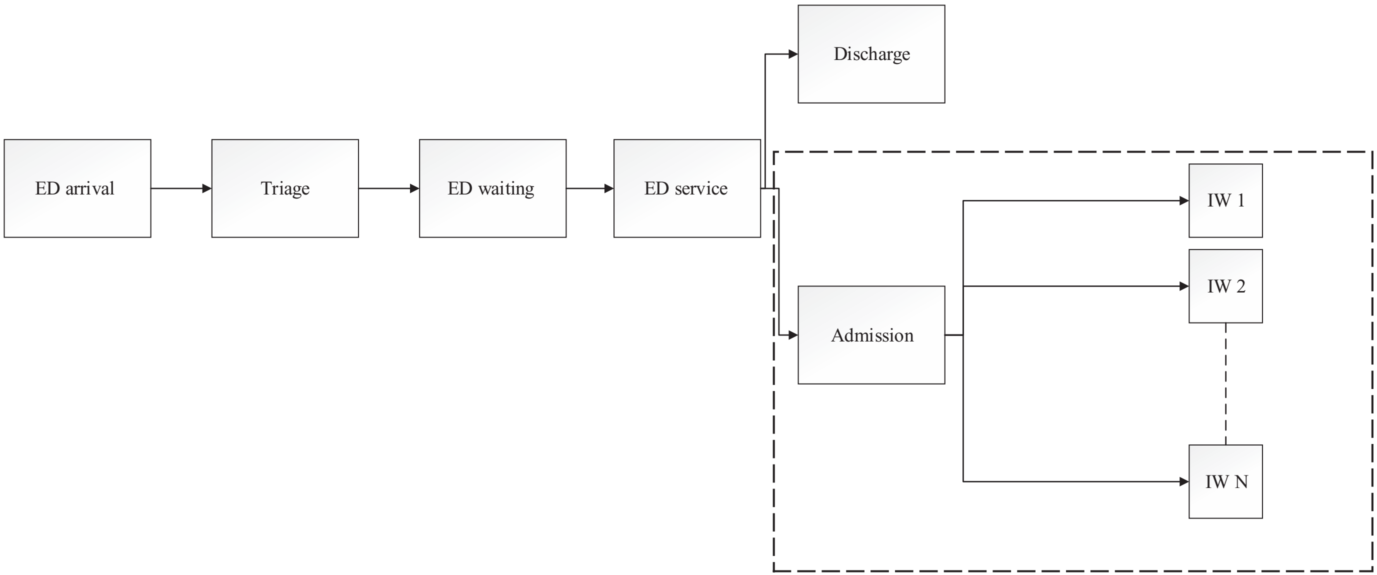


FIGURE 18.2 General flow of patients with the dotted area (inpatient ward, IW) representing the focus of this chapter

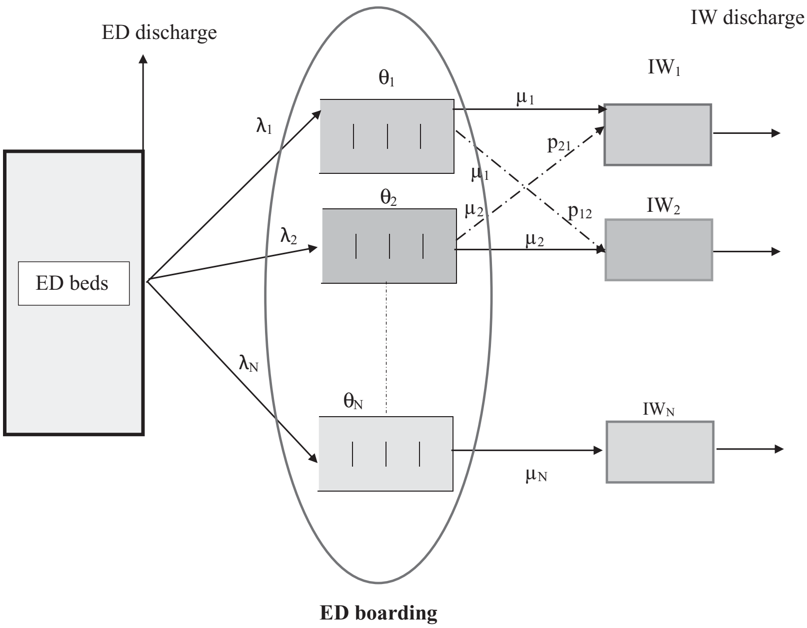


FIGURE 18.3 Queueing representation of the patient flow

Moreover, for $i \in N_p$ and $j \in N_s$, we let $a_{ij}(t) = 1$ if IW j is serving a class i patient at time t, and $a_{ij}(t) = 0$ otherwise.

Holding boarded patients in the ED is known to have negative consequences on their health status, including exposure to hospital-acquired infections, worse health outcomes, and even an increase in in-hospital death rates (see, e.g., Singer et al., 2011; Rabin et al., 2012; Sri-On et al., 2014). We term such negative consequences as "adverse events" and model their potential occurrence by class-dependent Poisson processes. In particular, we let $\bar{\theta}_i$ denote the per unit of time risk of adverse events (i.e., the rate of the underlying Poisson process) that can occur for a class i patient boarded in the ED, denote by c_i the associated cost per adverse event, define $\theta_i = c_i\bar{\theta}_i$, and let $\underline{\theta} = (\theta_i : i \in N_p)$. In this setting, θ_i plays the role of "expected holding cost" for a patient of class i and is accrued per unit of time boarding in the ED. Thus, keeping boarded patients in the ED for a longer amount of time is associated with a higher cost on average. However, the actual "holding cost" in our model is random and depends on stochastic deteriorations in the patient's conditions. Furthermore, in our setting, this cost metric varies for patients with different characteristics and conditions.[8]

[8] For simplicity and tractability, we model such stochastic deteriorations as a class-dependent stationary Poisson process (i.e., with a constant hazard rate for each patient class). This assumption helps us gain insights into the questions we raised in Section 18.1, especially because (1) estimation of time-dependent hazard rates from limited data is subject to significant errors and (2) the system dynamics will become non-Markovian under nonconstant hazard rates.

Similarly, for assignments to secondary inpatient units, we let p_{ij} denote the expected value of a nonnegative "penalty cost" (which is random in nature due to its dependency to various patient and provider-dependent conditions) that is accrued due to a lower-than-desired quality of care when a patient of class i is assigned to IW j ($p_{ij} = 0$ if $i = j$). This is supported by various studies in the literature that suggest overflowing patients to secondary units can be costly due to the mismatch between provider skills and patient conditions, which can lead to nonideal care and/or non-value added provider time because of increased travel between wards during rounds (see, e.g., Harrison et al., 2005; Gesensway, 2010; Rabin et al., 2012; Teow et al., 2012). For example, in Best et al. (2015), authors indicate that using lax IWs results in reduction in quality of care because nurses are not familiar with a wider range of care types. Additionally, the effectiveness of focused care on patient outcomes are highlighted in the literature by various studies (see, e.g., Kc and Terwiesch, 2011; Clark and Huckman, 2012).[9]

The objective is to find an optimal assignment policy to control the patient flow in order to minimize the expected total long-run average sum of (1) adverse events (a patient safety concern) and (2) the penalties accrued due to placement in secondary units (a quality of care concern).[10] This optimal objective can be calculated as

$$Z^* = \inf_{\pi \in \Pi} Z^{\pi} = \inf_{\pi \in \Pi} \left[\sum_{i \in N_p} \sum_{j \in N_s} p_{ij} O_{ij}^{\pi} + \sum_{i \in N_p} \theta_i L_i^{\pi} \right], \qquad (18.1)$$

where Π is the set of admissible (nonpreemptive, noncollaborative, and nonanticipative[11]) policies, Z^{π} is the long-run average objective under policy $\pi \in \Pi$, L_i^{π} denotes the long-run average number of class i patients in the queue (i.e., boarded in the ED) under policy $\pi \in \Pi$, and O_{ij}^{π} denotes the long-run average number of class i patients overflowed to IW j under policy $\pi \in \Pi$. In this setting,

$$L_i^{\pi} = \limsup_{T \to \infty} \frac{1}{T} \int_0^T E\left[X_i^{\pi}(s)\right] ds, \qquad (18.2)$$

$$O_{ij}^{\pi} = \limsup_{T \to \infty} \frac{A_{ij}^{\pi}(T)}{T}, \qquad (18.3)$$

9 In Section 18.6, we will discuss how we have used a year of data on patients with chest pain (CP) or congestive heart failure (CHF) to estimate all the parameters required for our model.

10 We may refer to these as costs for simplicity. However, it should be noted that these are general, as we discussed earlier, and may include various negative consequences of undesirable outcomes with respect to patient safety and/or quality of care caused by patient-flow decisions. We refer interested readers to empirical studies such as Kuntz et al. (2014), Berry Jaeker and Tucker (2016), Chan et al. (2016), and the references therein for further examples of such outcomes.

11 The reason we focus on nonanticipative policies is that even when the providers have a rough estimate on the discharge times of their patients, the exact discharge time is unknown and can be affected by several factors. Similarly, the exact timing of future bed requests are not known.

where $A_{ij}^{\pi}(T)$ is the cumulative number of times up to time T that IW j has been assigned to a class i patient under policy $\pi \in \Pi$ (i.e., a counting process associated with $a_{ij}(t) = 1$).

18.3.1 Markov Decision Process Formulation

As mentioned earlier, our partner hospital has eight main IWs (see Table 18.EC.1 in the online appendix), and hence, $|N_p| = |N_s| = 8$. However, each of these IWs only serve their primary and secondary patients. Therefore, the hospital network can conceptually be viewed as a collection of multiple primary-secondary IW pairs. Notably, insights gained from analyzing a single primary-secondary pair can be informative for the general assignment of ED boarded patients to their primary and secondary IWs. For this reason, and to gain some clear insights through analyzing a tractable model, we start by considering the simplest case where $N_p = N_s = \{1, 2\}$. We let $\underline{a_1} = (a_{11}, a_{21})$ and $\underline{a_2} = (a_{12}, a_{22})$, where $a_{ij} = 1$ if server j is busy with a patient of class i. We assume that all the underlying processes are memoryless and require that at any point in time $\sum_{i \in N_p} a_{ij} \leq 1 \ (\forall j \in N_s)$. To better reflect the actual practice, we mainly perform our analysis by assuming that preemptions are not allowed. In simple terms, preemption refers to interrupting the process of serving a customer or patient who is currently under service in order to serve another one (e.g, a customer or patient with a higher priority). In our setting, it might require transferring a patient from an IW back to the emergency room (or elsewhere) to use the IW bed for another patient.

With these, we define the system state as $\tilde{X} = (\underline{X}, \underline{a_1}, \underline{a_2})$ with state space $\mathcal{S} = \mathbb{Z}_+^2 \times \{0,1\}^2 \times \{0,1\}^2$.[12] We then use uniformization to transfer the underlying continuous-time Markov chain (CTMC) to a discrete-time Markov chain (DTMC). Let $\psi = \lambda_1 + \lambda_2 + 2\max\{\mu_1, \mu_2\}$ be the uniformization factor. Then, the long-run average cost optimality equation for the DTMC can be written as

$$J(\tilde{X}) + \hat{Z}^* = \frac{1}{\psi}\Bigg[\underline{\theta}\,\underline{X}^T + \min_{u=\underline{u_{ij}} \in \mathcal{U}(\tilde{X})} \Bigg\{ \sum_{i \in N_p} \sum_{j \in N_s} \lambda_i T^{\underline{u_{ij}}} J(\underline{X} + e_i, \underline{a_j}) + \sum_{i \in N_p} \sum_{j \in N_s} \sum_{k \in N_p} a_{kj}\mu_k T^{\underline{u_{ij}}} J(\underline{X}, \underline{a_j} - e_k) + \left(\psi - \sum_{i \in N_p} \lambda_i - \sum_{k \in N_p} \sum_{j \in N_s} a_{kj}\mu_k\right) J(\tilde{X}) \Bigg\}\Bigg], \qquad (18.4)$$

where $J(\tilde{X})$ is a relative cost function defined as the difference between the total expected cost of starting from state $\tilde{X}$ and a reference state (state $\underline{0}$), $\hat{Z}^*$ is the optimal average cost per uniformized period, the notation "^{T}" represents the

[12] Because we do not allow preemptions (to better reflect the actual practice), it is necessary to keep track of the servers' availabilities $(\underline{a_1}, \underline{a_2})$ as part of the state.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

transpose operator, and $T^{\underline{u_{ij}}}$ is a functional operator that depends on action vector $\underline{u_{ij}}$ and is defined in the online Appendix 18.B.

In optimality Eq. (18.4), e_i is a vector with the same dimensions as $\underline{X}$ containing a one in the ith position and zero elsewhere. Thus, the first line inside the minimization in Eq. (18.4) is due to inpatient bed request arrivals from the ED, which occur with rate λ_i for patients of class i. Similarly, the second line in Eq. (18.4) is due to discharges of patients from IWs, and the last line in Eq. (18.4) is due to the self-loop in the underlying DTMC. The control actions u_{ij} in Eq. (18.4) are taken so as to minimize the long-run average cost, where the set of admissible actions is

$$\mathcal{U}(\tilde{X}) = \Bigg\{ u = (u_{ij})_{i \in N_p, j \in N_s} \text{ s.t.} : u_{ij} \in \{0, 1\}, \sum_{i \in N_p} u_{ij} \leq \Big(1 - \sum_{i \in N_p} a_{ij}\Big) \ \forall j \in N_s, \sum_{j \in N_s} u_{ij} \leq X_i \ \ \forall i \in N_p \Bigg\}. \tag{18.5}$$

That is, a patient cannot be assigned to server j if server j is busy or if the number of patients boarded in the ED is insufficient. In the next section, we shed light on the optimal policy under the previously mentioned simplified setting.

In Section 18.6, we then relax various simplifying assumptions made (e.g., pooled beds in each IW, a single primary-secondary pair, and stationary arrival and service times) and test the validity of the insights gained in more complex scenarios using simulations calibrated with hospital data.

18.4 OPTIMAL ASSIGNMENT POLICY

In the online Appendix 18.C, we show that we can restrict our attention to policies that do not allow idling an IW $j \in N_s$ when there is a patient with IW j as their primary IW boarded in ED (see Proposition 18.EC.1 in Appendix 18.C).[13] Although this is an expected result in service systems in which preemption is allowed, we note that in nonpreemptive services such as the one we model, this insight can be counter intuitive. To establish this nonidling result under our nonpreemptive assumption, we first demonstrate a monotonicity property in Appendix 18.C (see Lemma 18.EC.1). Here, we seek to answer the questions we raised in the introduction, and generate insights into conditions under which patients should be forced to wait in the ED until a bed in their primary inpatient unit becomes available (rather than being transferred to a secondary unit with current bed availability). We start by establishing the following result.

Proposition 18.1 (Optimality of an Index-Based Priority Rule) *If $p_{ij} = 0$ for all $i \in N_p$ and $j \in N_s$, it is optimal for each IW to give strict priority to the patient class that has the highest $\theta_i \mu_i$ except to avoid idling, regardless of the status or allocation of other IWs.*

[13] Note that this result is only on idling when a primary patient exists and does not mean idling IW beds cannot be optimal in general (see, e.g., Theorem 18.1).

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

Strict priority rules are typically suboptimal in nonpreemptive service environments such as the one we study. Interestingly, however, Proposition 18.1 provides a sufficient condition under which inpatient units should give strict priority to serving the patient class with the highest $\theta\mu$ value: when reduction in quality of care is not a main concern, or similarly when the differences in service qualities between primary and secondary inpatient units are negligible. Labeling the class with the highest value of $\theta\mu$ as class 1, this means that although care delivery of patients cannot be preempted to accommodate a new bed request, in order to merely minimize the risk of adverse events, IWs should always prioritize serving class 1 patients when at least one such patient is boarded in the ED and the inpatient unit has some available capacity. The implication of Proposition 18.1 for a hospital bed manager is important and is as follows. If there is a class 1 patient boarded in ED that is not expected to experience a reduction in quality of care from an alternative IW assignment, the bed manager should prioritize assigning them to a bed as soon as one becomes available in either their primary or secondary IW: *patience is not a virtue* in this case.

But what if in addition to the risk of adverse events (a patient safety concern), the bed manager is also concerned about the quality of care? Our numerical results suggest that the optimal policy in such a situation is a state-dependent threshold-type policy, where the threshold is on the number of patients boarded in the ED. We will discuss this in detail in the remainder of this section. However, to gain some initial analytical insights, we first focus on the patient flow to IW 1. This allows us show that when we introduce nonzero overflow penalty costs in our model, the primary unit of class 1 patients (IW 1) prioritizes class 1 patients under the optimal policy whenever it has some available capacity and idles when $X_2 < \bar{X}_2$, where $\bar{X}_2$ is a threshold level. Thus, class 2 patients should be kept boarded in the ED rather than being overflowed to IW 1 when $X_2 < \bar{X}_2$. Hence, in this case, we find that patience is a virtue, but only up to a point.

> **Theorem 18.1 (Threshold-Based Idling)** *There exists an optimal stationary policy that is of a threshold type: IW 1 (1) serves its secondary patients when the number of such patients boarded in the ED reaches a state-dependent threshold level and has no primary patient boarded in the ED, (2) serves its primary patients whenever such patients are boarded in the ED, and (3) idles otherwise.*

As is specified in Theorem 18.1, it is optimal to idle IW 1 when there is no class 1 patient available and the number of class 2 patients waiting for an inpatient bed is below a threshold level. This is due to the nonzero overflow penalty, which represents the reduction in quality of care when a patient is assigned to a secondary unit. In fact, when p_{ij} is high enough, the optimal policy idles IW j whenever it does not have any primary patient boarded in the ED. For nonextreme overflow penalty cases, when IW 1 does not have a primary patient

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

boarded in the ED, it first idles until the number of class 2 patients boarded in the ED reaches a certain level and then prioritizes class 2 patients until either a class 1 patient starts to board in the ED, or the number of class 2 patients falls below the threshold. For hospital bed managers, Theorem 18.1 implies that when a bed becomes available in IW 1, class 1 patients should be assigned to that IW if there are class 1 patients boarded in the ED. Otherwise, class 2 patients should be assigned to IW 1, but only if the number of class 2 patients boarded in the ED is higher than a certain level. This insight is important because it sheds light on the fact that an IW 1 bed can be left idle under the optimal policy depending on the congestion level of the ED. By idling such a bed and asking class 2 patients to continue boarding in the ED, the hospital bed manager can avoid a potential reduction in quality of care and also prevent a future arriving class 1 patient from prolonged ED boarding, which in turn may have significant consequences regarding patient safety.

18.4.1 Patient Flow to IW 2

To gain further insights into the structure of effective patient-IW assignment policies, we now turn our attention to IW 2 and consider the simplified model illustrated in Figure 18.4. Recall that IW 2 is the primary IW for class 2 patients and the secondary IW for class 1 patients, where we labeled classes (without loss of generality) such that $\theta_1\mu_1 \geq \theta_2\mu_2$. Thus, IW 2 prefers to serve class 1 with respect to the $\theta\mu$ index, but class 2 with respect to the overflow penalty cost parameters. As we will see, understanding the main trade-offs in this simplified model is essential for answering the questions we raised in the introduction. Put differently, although the model presented in Figure 18.4 is a stylized version of the complex patient flow in hospitals, it allows us to gain useful insights that we can further test via realistic simulations (see Section 18.6).

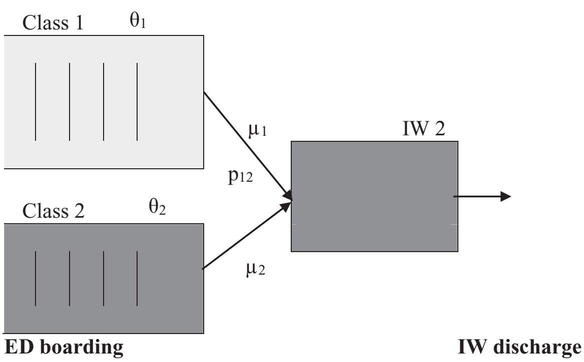


FIGURE 18.4 Queueing representation of the simplified system

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

We further simplify our analysis here by assuming that the service process is preemptive.[14] This allows us to consider $\underline{Y} = (Y_1, Y_2)$ as the system's state, where Y_i represents the number of class i patients in the system, the state space is $\mathcal{S} = \mathbb{Z}_+^2$, and the set of admissible actions is

$$\mathcal{U}(\underline{Y}) = \left\{ u = (u_{i2})_{i\in\{1,2\}} \text{ s.t.} : u_{i2} \in \{0,1\}, u_{i2} \le Y_i, \sum_{i\in\{1,2\}} u_{i2} \le 1 \;\; \forall i \in \{1,2\} \right\}. \tag{18.6}$$

Because the optimal policy and performance under long-run average setting can be obtained by using limit arguments over the infinite-horizon (see, e.g., Sennot, 1999), we start by considering the system in infinite horizon. The infinite-horizon optimality equation for this simplified model can be written as

$$\begin{aligned} J(\underline{Y}) = \underline{\theta}\, \underline{Y}^T + \beta \min_{u\in\mathcal{U}(\underline{Y})} \Bigg\{ &\sum_{i\in\{1,2\}} \tilde{\lambda}_i J(\underline{Y}+e_i) \\ &+ \sum_{i\in\{1,2\}} \tilde{\mu}_i u_{i2} \left(p_{i2} + J(\underline{Y}-e_i)\right) + \left(1 - \Lambda - \sum_{i\in\{1,2\}} \tilde{\mu}_i u_{i2}\right) J(\underline{Y}) \Bigg\}, \end{aligned} \tag{18.7}$$

where β is the discount factor per uniformized period, the overflow penalty cost parameters p_{12} and p_{22} are scaled so that $p_{22} = 0$, and the vector $\underline{\theta}$ is scaled so that $\underline{\theta}\, \underline{Y}^T$ represents the expected cost per uniformized period when the system is at state $\underline{Y}$. Moreover, in Eq. (18.7), the uniformization rate is $\bar{\psi} = \lambda_1 + \lambda_2 + \max\{\mu_1, \mu_2\}$, where $\tilde{\mu}_i = \frac{\mu_i}{\bar{\psi}}$, $\tilde{\lambda}_i = \frac{\lambda_i}{\bar{\psi}}$, and $\Lambda = \tilde{\lambda}_1 + \tilde{\lambda}_2$. Next, we define the functional operators T_a, T_u and T_* (see, e.g, Saghafian and Veatch, 2016) for the use of similar operators in a different queueing structure) as

$$T_\theta J(\underline{Y}) = \underline{\theta}\, \underline{Y}^T, \tag{18.8}$$

$$T_a J(\underline{Y}) = \sum_{i\in\{1,2\}} \tilde{\lambda}_i J(\underline{Y}+e_i), \tag{18.9}$$

$$\begin{aligned} T_u J(\underline{Y}) &= \sum_{i\in\{1,2\}} \tilde{\mu}_i u_{i2} \left(p_{i2} + J(\underline{Y}-e_i)\right), \left(1 - \Lambda - \sum_{i\in\{1,2\}} \tilde{\mu}_i u_{i2}\right) J(\underline{Y}), \\ &= (1-\Lambda) J(\underline{Y}) - \sum_{i\in\{1,2\}} \tilde{\mu}_i u_{i2} \left(\Delta_i J(\underline{Y}-e_i) - p_{i2}\right), \end{aligned} \tag{18.10}$$

[14] We realize that allowing service preemption is not fully realistic; however, this assumption is useful for tractability and for gaining sharp insights. We relax this assumption in Section 18.6 and utilize real-world data along with simulation analyses to verify the insights gained.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

$$T_*J(\underline{Y}) = \min_{u \in \mathcal{U}(\underline{Y})} T_u J(\underline{Y}), \tag{18.11}$$

$$TJ(\underline{Y}) = T_\theta J(\underline{Y}) + \beta\left(T_a J(\underline{Y}) + T_* J(\underline{Y})\right), \tag{18.12}$$

where $\Delta_i J(\underline{Y}) = J(\underline{Y}+e_i) - J(\underline{Y})$. Using these functional operators, we can simply write the infinite-horizon optimality Eq. (18.7) as

$$J(\underline{Y}) = TJ(\underline{Y}). \tag{18.13}$$

The average cost and finite-horizon cost equations can be obtained in a similar manner. Specifically, the finite-horizon cost satisfies $J_{n+1}(\underline{Y}) = TJ_n(\underline{Y})$, and the average cost can be calculated as $\lim_{\beta \to 1^-}(1-\beta)J(\underline{Y})$ (see, e.g., Sennot, 1999 Corollary 7.5.10 for further discussion).

Using the previsouly mentioned setting, we next consider the following two properties:

(*i*) $\mu_1 \Delta_1 J(\underline{Y}) - \mu_2 \Delta_2 J(\underline{Y} + e_1 - e_2) \geq \mu_1 \Delta_1 J(\underline{Y} - e_1) - \mu_2 \Delta_2 J(\underline{Y} - e_2)$ for all $\underline{Y} \geq (1,1)$, (18.14)

(*ii*) $\mu_1 \Delta_1 J(\underline{Y} - e_1) - \mu_2 \Delta_2 J(\underline{Y} - e_2) \geq \mu_1 \Delta_1 J(\underline{Y} + e_2 - e_1) - \mu_2 \Delta_2 J(\underline{Y})$ for all $\underline{Y} \geq (1,1)$. (18.15)

Property (1) implies that assigning class 1 patients to IW 2 becomes more desirable as the number of boarded class 1 patients increases, and property (2) implies that assigning class 2 patients to IW 2 becomes more desirable as the number of boarded class 2 patients increases.

Let $\mathcal{F}$ be the set of real-valued functions defined on $\mathcal{S} = \mathbb{Z}_+^2$ such that if $F \in \mathcal{F}$ then F satisfies properties (18.14)–(18.15). The following lemma shows that, if $\theta_1\mu_1 \geq \theta_2\mu_2$, the functional operator T defined in Eq. (18.12) preserves properties (18.14)–(18.15).

Lemma 18.1 (Preservation) *If* $\theta_1\mu_1 \geq \theta_2\mu_2$ *and* $J \in \mathcal{F}$*, then* $TJ \in \mathcal{F}$.

By making use of Lemma 18.1, we can establish the following result.

Theorem 18.2 (Optimality of a Threshold-Type Policy) *If* $\theta_1\mu_1 \geq \theta_2\mu_2$*, then the optimal policy obtained from Eq. (18.7) is of a threshold type: IW 2 should prioritize its primary patients until the number of class 1 patients boarded in the ED reaches a threshold that depends on the number of class 2 patients still waiting for a bed assignment.*

The optimal policy described in Theorem 18.2 is a threshold-based primary-then-$c\mu$ rule: IW 2 serves its primary patients up to a point and switches to the $c\mu$ rule ($\theta\mu$ in our notation) afterward. Note that when $p_{12} = 0$, the optimal assignment policy is the well-known $c\mu$ rule (see, e.g., Buyukkoc et al., 1985; Saghafian and Veatch, 2016), because the threshold becomes zero. However, when we consider a nonzero penalty cost in the model, under the optimal policy, IW 2 first serves its primary patients until the marginal benefit of serving a primary patient versus a secondary one reaches the value of the penalty that might be accrued due to the reduction in quality of care. This suggests that, when the number of boarded patients is low, EDs should try to match their patients with their primary units to ensure the highest quality of care. However, once the number of boarded patients passes a specific threshold, the focus should shift from concerns about decrements in quality of care to concerns about the risk of adverse events that can occur due to prolonged boarding. Thus, we again observe that *patience (for a primary unit assignment) is a virtue, but only up to a point.*

Hospital bed managers can use our results in various ways when deciding on which patient class to assign to an IW 2 bed that has just become available. For example, when a bed becomes available in IW 2, and they do no expect any near-term bed availability in IW 1, Theorem 18.2 suggests that bed managers should consider the number of both class 1 and 2 patients boarded in ED and prioritize the primary patient type (class 2) until the number of class 1 patients boarded in ED reaches a certain level. From then on, they should start prioritizing class 1 patients until the number of class 1 patients boarded in ED drops below that certain level. However, the bed manager should be aware that this level is highly dependent on the number of patients from both classes in the ED as well as estimation of parameters related to (1) reduction in quality of care when a secondary inpatient unit is used (p_{ij}), (2) risk of adverse events for both classes (θ_i), and (3) average length of stay for both classes ($\frac{1}{\mu_i}$). Thus, the decision should be made in a careful way and only after performing sensitivity analysis.

To further assist hospital bed managers in making such decisions, we utilize the insights we gained from analyzing the optimal policy of our simplified models and develop effective bed assignment heuristics in the next section. We then use a variety of realistic simulation experiments (calibrated with hospital data that we have collected) to evaluate the effectiveness of proposed bed assignment policy under realistic conditions and generate more detailed insights for hospital administrators via sensitivity analyses. By making use of our realistic simulation experiments, we also test the validity of the insights we gained in this section by analyzing tractable though stylized and simplified representations of the patient flow.

18.5 HEURISTIC POLICIES

When we consider nonpreemptive service policies (which better represent the current practice in most hospitals) under the general system structure discussed in Section 18.3, our numerical computations show that the optimal policy is complex: It has a state-dependent threshold that depends on all the elements in the system state, including IW bed availabilities. Our numerical results also show that the optimal policy has a structure similar to the optimal control of

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

the N structure queueing network, where one server works as a shared server, whereas the other works as a dedicated server. That is, the primary unit of class 1 patients (IW 1) typically prioritizes its primary patients (i.e., works as a dedicated unit whenever its queue of boarded patients is not empty), and primary unit of class 2 patients (IW 2) typically first serves its primary patients until the number of class 1 patients boarded in ED exceeds a threshold, then helps IW 1 by serving class 1 patients (see Appendix 18.C for some numerical experiments supporting this observation). In what follows, we take advantage of this (as well as our earlier findings) to develop easy-to-implement heuristic policies for use in hospitals.

18.5.1 Birth-and-Death Process to Approximate the Optimal Threshold

To develop a heuristic that is easy to implement, we start by considering the optimal policy of an N queueing network by assuming that IW 2 can serve patients from both types (a shared server) while IW 1 can only serve class 1 patients (a dedicated server). We use a birth-and-death process for this system to estimate the optimal threshold level on the number of class 1 patients boarded in the ED above which IW 2 starts helping IW 1 by serving class 1 patients. In particular, assuming that the threshold level is some number T, we can approximate the class 1 queueing dynamics via the birth-and-death process (see Figure 18.EC.6 in Appendix 18.D). When the number of patients in the class 1 queue, X_1, is smaller than the threshold level, only IW 1 will serve class 1 patients, which will occur with rate μ_1. However, when X_1 is larger than the threshold, both IW 1 and IW 2 will serve class 1 patients, and hence, the death rate becomes $2\mu_1$.

We use a separate birth-and-death process to approximate the dynamics for class 2 patients (see Figure 18.EC.7 in Appendix 18.D). Let $P^2(T)$ be the steady-state fraction of time that IW 2 serves class 2 patients. Then, the service rate for class 2 patients is $P^2(T)\mu_2$. Let $L^1(T)$ and $L^2(T)$ denote the long-run average queue length (i.e., number of patients boarded in the ED) of class 1 and 2 patients, respectively. Assuming that $O^1(T)$ denotes the average number of class 1 patients served by IW 2 under threshold T, and $Z(T)$ denotes the long-run average system's cost under threshold T, we can calculate $Z(T)$ as

$$Z(T) = \theta_1 L^1(T) + \theta_2 L^2(T) + p_{12} O^1(T). \qquad (18.16)$$

The objective is to find the value of T that minimizes $Z(T)$. To calculate Eq. (18.16), we use the birth-and-death processes to estimate $L^1(T)$, $L^2(T)$, and $O^1(T)$. To this end, we first need to obtain the steady-state probability P_i^j, which is the probability that the length of queue $j \in \{1, 2\}$ equals $i \geq 0$. From the balance equations, we have

$$P_i^1 = \left(\frac{\lambda_1}{\mu_1}\right)^i P_0^1 \qquad \forall\ i \leq T, \qquad (18.17)$$

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

$$P_i^1 = \left(\frac{\lambda_1}{\mu_1}\right)^T \left(\frac{\lambda_1}{2\mu_1}\right)^{i-T} P_0^1, \quad \forall\ i > T. \tag{18.18}$$

By using the fact that these probabilities must sum to one, we find P_0^1 as

$$P_0^1(T) = \frac{(1-\rho_1)(1-\rho_2)}{\rho_1^T(\rho_2-\rho_1)+(1-\rho_2)}, \tag{18.19}$$

where $\rho_1 = \frac{\lambda_1}{\mu_1}$ and $\rho_2 = \frac{\lambda_1}{2\mu_1}$. By using these probabilities, we can obtain the average queue length for class 1 patients, $L^1(T)$:

$$L^1(T) = \sum_{i=0}^{T} i\rho_1^i P_0^1(T) + \sum_{i=T+1}^{\infty} i\rho_1^T \rho_2^{i-T} P_0^1(T). \tag{18.20}$$

Also, $O^1(T) = \frac{1}{2}\sum_{i=T+1}^{\infty} i\rho_1^T\rho_2^{i-T}P_0^1(T)$ by assuming that class 1 patients in the queue will be served equally by IW 1 and IW 2 after the number of class 1 patients boarded in ED reaches the threshold.[15] To calculate the average queue length of class 2 patients, $L_2(T)$, we first calculate the following:

$$P^2(T) = P(x_1 \le T) = P_0^1(T)\frac{1-\rho_1^{T+1}}{1-\rho_1}. \tag{18.21}$$

The average queue length for class 2 patients is then

$$L^2(T) = \frac{\lambda_2}{P^2(T)\mu_2 - \lambda_2}. \tag{18.22}$$

These allow us to calculate $Z(T)$ via Eq. (18.16), and find the optimal threshold value $T^* = \arg\min_{T\ge 0} Z(T)$. However, the threshold level T^* does not have a closed-form solution, and the function $Z(T)$ can be nonconvex in general. Nevertheless, we can utilize numerical approaches (e.g., bisection search) to find the value that minimizes Eq. (18.16). We term the heuristic policy that controls the patient flow based on this threshold as the birth-and-death threshold (BDT) policy.

18.5.2 Penalty-Adjusted Largest Expected Workload Cost Policy (LEWC-p)

Our results in Section 18.4 reveal that there exists a threshold-type optimal policy that optimizes performance by following the primary-then-$c\mu$ rule (see, e.g., Theorem 18.2). This policy tends to serve the primary patient type with the lower $c\mu$ value until the cost differences of serving the secondary patients exceeds the overflow penalty cost (see the discussion in Appendix 18.C, proof

[15] This is not a strong assumption because the service rates are patient class dependent, not IW dependent.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

of Lemma 18.1). This insight suggests that instead of using a heuristic policy to directly approximate the threshold – the idea behind the BDT policy – there might be value in following a heuristic that balances the costs associated with different queues. Thus, as our second heuristics, we develop a modified version of the largest expected workload cost (LEWC) policy proposed by Saghafian et al. (2011) for general parallel queueing systems.

The LEWC policy dynamically balances the expected workload cost of queues by prioritizing the queue with the largest expected workload cost (ROAE in our setting).[16] In order to also incorporate the additional penalty cost of serving patients in their secondary IW – a main factor affecting the patient flow – we propose a penalty-adjusted version of LEWC, which we term LEWC-p. To this end, similar to Saghafian et al. (2011), we first use the following linear program (LP). In this LP, the objective is to find the optimal server allocations to maximize the minimum percentage excess capacity among all patient types:

$$\text{Max} \quad \tau \tag{18.23}$$

Subject to

$$\sum_{j \in N_s} y_{ij}\mu_i \geq \lambda_i(1+\tau) \qquad \forall i \in N_p, \tag{18.24}$$

$$\sum_{i \in N_p} y_{ij} \leq 1 \qquad \forall j \in N_s, \tag{18.25}$$

$$y_{ij} \geq 0 \qquad \forall i \in N_p, \forall j \in N_s. \tag{18.26}$$

In this LP, y_{ij} is the decision variable that represents the long-run proportion of time that IW j serves patient class i. Constraint (18.24) ensures that the objective function maximize the minimum excess capacity among all patient classes. Constraint (18.25) guarantees that the total proportion of time for each IW does not exceed one, and Constraint (18.26) enforces the proportions to be non-negative.

Next, when a bed in IW j becomes available, we calculate an index, $I_{ij}(x_i)$, for each queue $i \in N_p$ (class of patients boarded in the ED) to approximate the penalty-adjusted expected workload cost of that queue given that its current length is x_i:

$$I_{ij}(x_i) = \frac{\theta_i x_i}{\sum_{j \in N_s} y^*_{ij}\mu_i} - p_{ij}\frac{x_i y^*_{ij}}{\sum_{j \in N_s} y^*_{ij}}, \tag{18.27}$$

where y^*_{ij}'s are the solution to LP (18.23)–(18.26). The first part of the index approximates the cost associated with risk of adverse events for class i patients: Because there are x_i patients in the queue, it will take approximately $\frac{x_i}{\sum_{j \in N_s} y^*_{ij}\mu_i}$ units of time to serve them, and the cost due to adverse events is θ_i per unit

[16] LEWC is a dynamic policy because it prescribes different actions based on the system state.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

of time per patient boarded. The second part of the index approximates the associated penalty cost. In this term, $\frac{y_{ij}^*}{\sum_{j\in N_s} y_{ij}^*}$ represents the proportion of patients of class i served by IW j.[17]

With these, the penalty-adjusted LEWC policy (LEWC-p) is as follows:

Step 1. Solve LP (18.23)–(18.26) to derive optimal allocations y_{ij}^*.

Step 2. Whenever a patient arrives or IW j becomes available, compute indices $I_{ij}(x_i)$ for all patient classes ($i \in N_p$), then assign the bed to patient class $k = \arg\max_{i\in N_p} I_{ij}(x_i)$. If the primary and secondary queues of IW j have the same index, break the tie by assigning the bed to the primary queue. If the primary queue of IW j is empty, and its secondary queue has a negative index, keep the bed in IW j idle.

Note that LEWC-p considers both the expected ROAE associated with number of patients that are boarded in ED and the penalty cost due to assignment to the secondary IW. Therefore, LEWC-p allows a decision-maker to assign patients to their secondary IWs only when the marginal benefit of serving these patients is greater than the penalty cost that will be accrued due to the reduction in quality of care. Hence, employing LEWC-p helps us to reflect the insights that we gained from analyzing the optimal policy through an easy to implement heuristic policy. Furthermore, our analyses of the optimal policy allows us to compare the performance of this heuristic policy vis-a-vis the optimal one (for small, tractable settings).

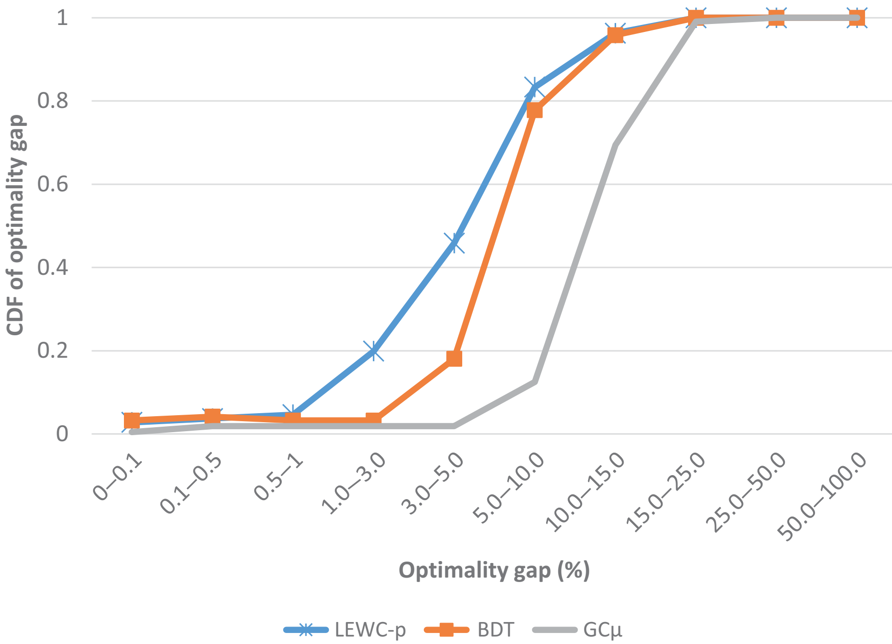
18.5.3 Comparison of the Proposed Heuristic Policies

We now compare the performance of the proposed BDT and LEWC-p heuristic policies with the optimal policy. As a benchmark, we also use the generalized $c\mu$ (G$c\mu$) rule. Under the G$c\mu$ policy, the available bed in IW j is assigned to the class that has the highest $\theta_i\mu_i x_i$ value. We use this policy as a benchmark since it (1) takes the queue lengths into account and (2) is known to work well in a variety of queueing systems.[18]

To compare these policies (BDT, LEWC-p, G$c\mu$, and optimal), we create a large test suite that covers various combinations of parameters (e.g., costs associated with risk of adverse events and reduction in quality of care, arrival rates, and service rates). Tables 18.EC.5–18.EC.7 in Appendix 18.E summarize

[17] In using Eq. (18.27), we assume that LP (18.23)–(18.26) has a unique optimal solution with $y_{ij}^* \neq 0$ whenever $i \neq j$. For systems in which this solution is not unique (e.g., balanced systems where $\frac{\lambda_i}{\mu_i} = \kappa, \quad \forall i \in N_p$), ties need to be broken based on cost parameters.

[18] This is especially the case in systems with quadratic holding costs and in systems that face heavy traffic. Our system does not meet any of these conditions. However, we make use of the optimality gap of the G$c\mu$ rule to better gauge the optimality gap of our proposed heuristics.



	Optimality gap			
Policy	Mean	Std.dev.	Min	Max
LEWC-p	6.19%	3.94%	0.00%	16.54%
BDT	7.97%	4.17%	0.00%	28.38%
GCμ	13.41%	4.64%	0.44%	34.25%

FIGURE 18.5 Performance of LEWC-p, BDT, and $Gc\mu$ relative to the optimal policy over the entire test suite (216 problem instances)

the parameter combinations in this test suite, which generate a total of 216 problem instances. To find the optimal policy for each problem instance, we use the well-known value-iteration algorithm to solve our MDP formulation. This allows us to report optimality gaps for each of the policies under consideration.

Figure 18.5 illustrates our computational results over the test suite by constructing the empirical cumulative distribution function (CDF) for the percentage optimality gap of each of the non-optimal policies (BDT, LEWC-p, and G$c\mu$). The results presented in this figure show that LEWC-p and BDT policies can both be considered as nearly optimal policies. However, the mean and standard deviation of LEWC-p optimality gap is smaller than that of the BDT policy, so we can conclude that LEWC-p is the better policy. The performance of $Gc\mu$ is, however, significantly worse than both the LEWC-p and BDT policies. This is mainly because $Gc\mu$ does not consider penalties associated with secondary unit assignments. However, even when the underlying penalty

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

TABLE 18.1 *Optimality gap of policies for various congestion levels*

Congestion level	Policy	Mean	Min	Max
Low: $\rho \leq 0.5$	LEWC-p	7.01%	0.00%	14.03%
	BDT	8.66%	0.00%	28.38%
	Gcμ	16.90%	11.90%	34.25%
Moderate: $\rho = 0.7$	LEWC-p	5.83%	1.29%	16.54%
	BDT	8.34%	2.61%	15.47%
	Gcμ	13.47%	8.66%	17.17%
High: $\rho \geq 0.9$	LEWC-p	5.68%	1.74%	9.20%
	BDT	7.07%	3.82%	14.42%
	Gcμ	10.49%	0.09%	15.28%

parameter is zero, we observe that $Gc\mu$ is not the best policy for all cases. When the penalty parameter is zero, both of the proposed heuristic policies (BDT and LEWC-p) perform close to each other, whereas BDT performs slightly better due to the assumption that IW 1 only serves class 1 patients (under the $c\mu$ policy, both of the IWs serve class 1 whenever feasible).

Table 18.1 compares the optimality gap of LEWC-p, BDT, and $Gc\mu$ policies for various congestion levels in the system. All of the policies show a smaller mean optimality gap in moderate to high congestion levels than in the low congestion level. This observation suggests that implementing them in crowded systems (e.g., in busy teaching hospitals) is better than doing so in less crowded systems (e.g., in less busy urban hospitals). Finally, Table 18.2 compares the policies based on various penalty parameter settings and shows that all policies perform best when the underlying penalty parameter is high. Moreover, LEWC-p is more robust than the BDT policy to changes in the penalty parameter. This is intuitive because the BDT policy only uses one threshold level, whereas the LEWC-p policy dynamically adjusts the assignments based on the number of patients of different classes that are boarded in the ED. Overall, under various congestion and penalty parameter settings, we observe that LEWC-p outperforms the BDT and $Gc\mu$ policies.

18.6 SIMULATION ANALYSIS USING HOSPITAL DATA

To gain more insights into effective policies for assigning ED patients to their primary or secondary inpatient units, we use a discrete-event simulation model of ED patient flow and calibrate it with a year of hospital data that we have collected from our partner hospital. This enables us to relax some of the assumptions we made earlier (e.g., exponential service times and Poisson arrivals), and also shed light on the magnitude of achievable benefits for EDs as well as hospital conditions under which our proposed assignment

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

TABLE 18.2 *Optimality gap of policies for various penalty cost parameters*

Penalty cost	Policy	Mean	Min	Max
Low: $p_{12} = p_{21}$=1	LEWC-p	7.02%	1.29%	16.54%
	BDT	9.87%	6.00%	28.38%
	Gcμ	14.37%	8.85%	21.78%
Moderate: $p_{12} = p_{21}$=10	LEWC-p	5.82%	0.00%	15.06%
	BDT	6.17%	0.00%	15.73%
	Gcμ	12.92%	0.09%	22.40%
High: $p_{12} = p_{21}$=100	LEWC-p	4.68%	0.00%	13.24%
	BDT	4.68%	0.00%	13.92%
	Gcμ	11.73%	0.44%	34.25%
Low-high: $p_{12} = 1, p_{21}$=100	LEWC-p	4.55%	2.78%	4.73%
	BDT	5.21%	4.59%	7.24%
	Gcμ	11.68%	10.49%	13.05%
High-low: $p_{12} = 100, p_{21}$=1	LEWC-p	7.96%	5.74%	10.15%
	BDT	10.22%	8.74%	12.53%
	Gcμ	15.42%	11.14%	16.41%

policy (LEWC-p) will work well. To this end, we first describe the admission sources in our partner hospital. We then describe the arrival process from such sources. Finally, we discuss the service process as well as the empirical length of stay (LOS) distributions and other parameters that we have estimated from our data set.

18.6.1 Patient Flow and IWs in Our Partner Hospital

18.6.1.1 Admission Sources

Patients are admitted to IWs from three main sources. We categorize admitted patients based on their source of admission in three groups: ED admits, direct admits, and operating room (OR) admits. ED admits are patients who finish their treatment with ED and receive an admit decision from an ED physician. Direct admits are the ones directly admitted to an IW without any preceding visits, and OR admits are the patients who initially receive a surgery from the hospital and are subsequently admitted to an IW.

18.6.1.2 IWs

Patients from these three admission sources require a bed from one of the eight inpatient units based on their diagnosis. The name of IWs, their descriptions, and number of beds in each of them in our partner hospital can be found in Table 18.EC.1 in the online appendix (see Section 18.1).

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

18.6.1.3 Patient Types

To gain clear insights into effective assignment policies, we focus on patients who were admitted via the ED of our partner hospital with an admission diagnosis of either chest pain (CP) or congestive heart failure (CHF). The main reason why we focus on these patients is data availability. We have access to complete and reliable data on time of bed requests, inpatient LOS, and assigned IW for CP and CHF patients (but not other patients). Moreover, these patients are often assigned to a secondary IW; the primary IW for both CP and CHF patients is 4 West (4W), and their secondary IW is 5 West (5W) (see Table 18.EC.1 for more information regarding these IWs).

There are two types of CP and CHF patients: Type 1 patients are those considered to be more sensitive to a secondary bed assignment (i.e., are subject to higher reduction in quality of care if assigned to a secondary inpatient unit). Type 2 patients are those who are less sensitive to a secondary bed assignment. We developed a classification scheme using simple laboratory findings and based on our discussions with medical experts at our partner hospital. We define Type 1 CP patients as those who have an elevated serum troponin (Tn) level, and type 2 CP patients as those who have a normal troponin level. We define type 1 CHF patients as those who have a B-type natriuretic peptide (BNP) level of 4,000 pg/ml or greater, and type 2 CHF patients as those with BNP levels below 4,000 pg/ml. Our empirical analyses show that, among patients of same type, there is no statistically significant difference in the mean IW service time between primary and secondary units (see Table 18.EC.8 in Appendix 18.F).

18.6.1.4 Arrival Process

We use bed-request times as the arrival time of each patient to our system. We observe from our data set that, for each of the three arrival sources (ED admit, direct admit, and OR admit), the arrival rate is highly time-dependent. Furthermore, we observe that the arrival process for each arrival source and for each IW can be modeled as a nonhomogeneous Poisson process with a rate that is constant during one-hour time blocks. In addition to hour-of-day dependent arrival rates, we observe day-of-week dependency in arrival rates for ED admits. We simulate the patient flow assuming that the arrival process is cyclo-stationary with one week as the cycle. We do not consider the rare transfers between inpatient units because (1) our focus in this chapter is on the patient flow between ED and IWs, (2) these transfers do not have any significant effect on the optimal policy, and (3) based on our data set, the rate of such transfers is negligible compared to the arrival rate of ED admits, direct admits, and OR admits.

18.6.1.5 Service Process

In our simulation model, we consider the beds in IWs as servers. Based on our data, the service rates depend on patient type and admission source but not the IW (see Table 18.EC.8 in Appendix 18.F for p-values on the equality of means of service times for primary and secondary IWs for different patient types). Table 18.EC.9 in Appendix 18.F shows the average service time (in days) for

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

each IW based on the admission source. Our statistical analyses suggest that we can use lognormal distributions as service time distributions.[19]

18.6.1.6 Costs

Penalty costs are assigned based on the patient type (types 1 and 2 discussed earlier). The average penalty cost for type 1 patients are always higher than that of type 2 patients because type 1 patients are more sensitive to a secondary bed assignment. However, due to the lack of data on quality of care and patient safety, estimating cost parameters is inherently subject to error and thus necessitates performing various sensitivity analyses. To perform such sensitivity analyses, we consider a range of parameters for both penalty costs and costs associated with risk of adverse events (see Appendix 18.G for more information). This range of parameters are provided by our physician collaborators and are intended to represent values that are realistic while covering possible differences among hospitals. Of note, the cost parameters that we use in our analyses are also chosen based on the medical characteristics of patients. For example, to determine the appropriate range of cost parameters related to the risk of adverse events (ROAE), we consider the emergency severity index (ESI) of patients boarded in the ED because ESI is a direct indicator of urgency. Similarly, our physician collaborators determined the appropriate range of penalty cost parameters by using patients' chief complaints (among other patient characteristics) and by ensuring that they are comparable with the range considered for ROAE cost parameters. Although we rely on cost estimations provided by our physician collaborators and do not provide a formal method to directly estimate them, we perform various sensitivity analyses and study the performance of our proposed policies under different scenarios.

18.6.1.7 Performance Measures

In addition to the overall objective we introduced in Section 18.3, we use the overflow proportion (the ratio of patients assigned to a secondary IW to the total number of patients of same type served) and the average ED boarding time (the average time between a request and bed occupancy) as other performance measures. We also use the two-hour boarding rate (the fraction of patients that are boarded for two hours or more)[20] as another performance measure. We do so because reducing excessive boarding times (and not just average boarding times) is also important for most EDs.

18.6.1.8 Priorities and Runs

We use the first-in-first-out (FIFO) priority rule for each IW regardless of the admission source of patients. Each simulation observation is obtained for 1,000 replications with a replication length of one year. The number of replications is

[19] Log-normal distribution as a service time distribution is not unique to hospitals. For instance, Brown et al. (2005) show similar characteristic of the service time distribution in call centers.

[20] As we discussed in Section 18.1, the current two-hour boarding rate at our partner hospital based on our data set is around 30%.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

chosen so as to enforce tight confidence intervals, enabling us to represent simulation confidence intervals with their midpoint in all of our graphs. This warm-up period is determined through the Welch method (see, e.g., Welch, 1983).

18.6.1.9 Base Case Scenario

We consider the base case scenario to be a reflection of the current system in our partner hospital based on a year of data that we have collected. We use this scenario as a benchmark to analyze the potential changes that may occur due to implementing our proposed policies. Thus, we use the level of performance measures in the base case scenario (e.g., two-hour boarding rate, average boarding time in the ED, etc.) for CP and CHF patients as a point of reference and compare the results of our proposed policy with those metrics. To this end, we focus on patient flow from the ED to the two IWs that can serve CP and CHF patients: 4 West and 5 West. In addition to CP and CHF patients, we simulate the flow of other patients that require a bed from 4 West or 5 West but note that these patients are not eligible for overflows and can only be assigned to their primary inpatient units. We include these patients in our simulation model to represent the capacity utilization in 4 West and 5 West more accurately, thereby increasing the fidelity of our simulations.

Figure 18.6 illustrates the patient flow under consideration. Current data availability does not allow us to model the flow in the entire hospital network and test the performance of proposed heuristic policy in a larger network. Thus, here we stay with CP and CHF patients (i.e., patients for which we

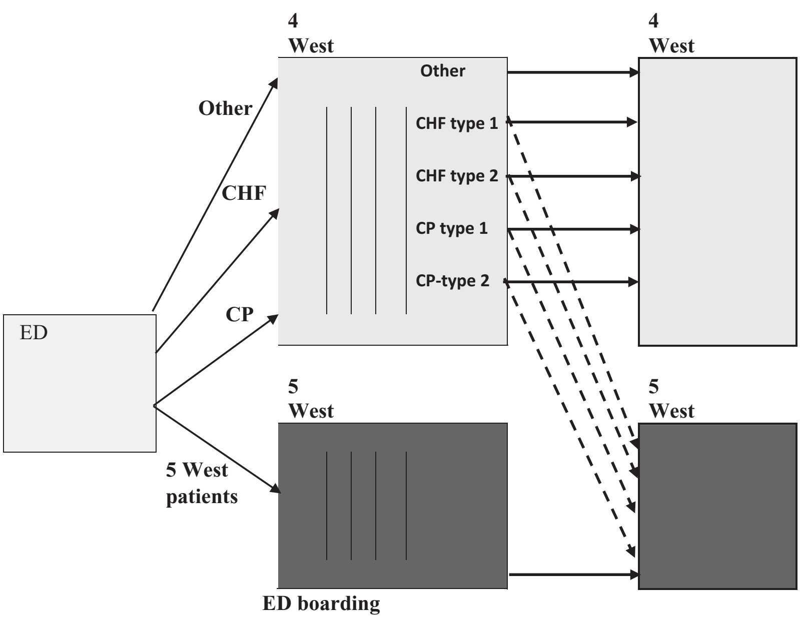


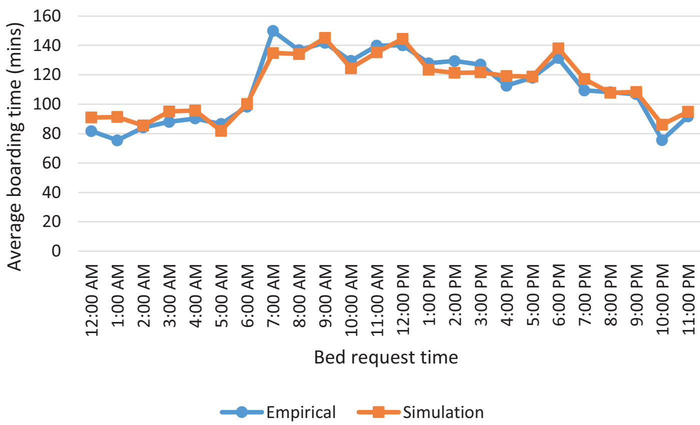
FIGURE 18.6 Patient flow in the simulation model for CHF and CP patients

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

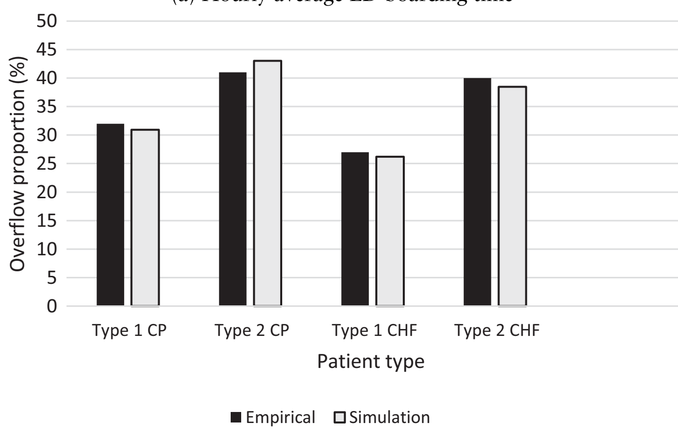
have complete data) and only consider their destinations, as shown in Figure 18.6 (see Appendix 18.H, where perform simulations considering other hospital IWs). The dashed lines in Figure 18.6 show assignments of patients to secondary IWs (overflows that incur a penalty cost), whereas the solid lines show assignments to primary IWs. In the current practice, there is no specific rule for assigning patients to their primary versus secondary units. Thus, for our base case scenario, we use the FIFO rule for the primary bed assignments and take a data-driven approach to represent the overflows to secondary IWs by using the proportions that are obtained from our data analyses.

18.6.2 Validating the Simulation Model

To validate our simulation model, we compare our empirical results obtained directly from our data set with those obtained from our simulation model. Figures 18.7(a) and 18.7(b) compare the resulting time-dependent boarding



(a) Hourly average ED boarding time



(b) Overflow proportion

FIGURE 18.7 Validating the simulation model

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

time of patients as well as the resulted overflow rates of the simulation model with that of the empirical data. Using the t-test for the equality of means, we observe no statistical difference between outputs of our simulation model and those from empirical data (p-value = 0.412). Similarly, using Kolmogorov-Smirnov tests for comparing the distributions of outputs (e.g., boarding time distributions) with the empirical distributions from our data, we do not observe any significant mismatch. These results give us confidence that our simulation model is relatively of high fidelity and accurately matches the current practice.

18.6.3 Performance of the Proposed LEWC-p Policy

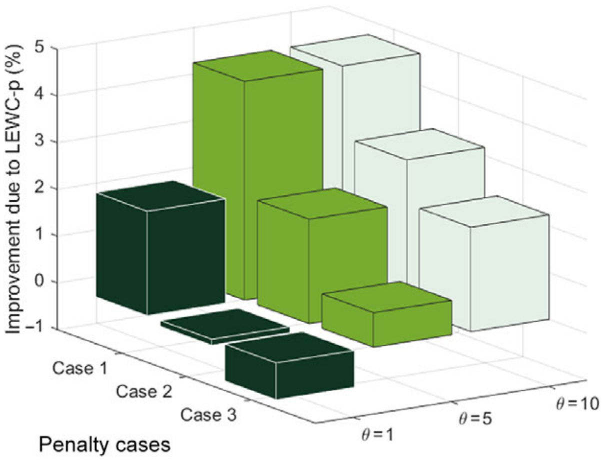
We now use our simulation model for CP and CHF patients to investigate the impact of implementing our proposed LEWC-p policy. Based on our results, we make the following observation:

> **Observation 18.1 (Benefits of LEWC-p)** *Implementing LEWC-p for assigning CP and CHF patients to their IWs improves the total average cost by 14%, the two-hour boarding rate by 2%, and the average boarding time by 9% (10 minutes/patient). Also, compared to current practice, these improvements due to implementing LEWC-p are all statistically significant (the p-value on the difference is 0.00018, 0.022, and 0.001, respectively).*

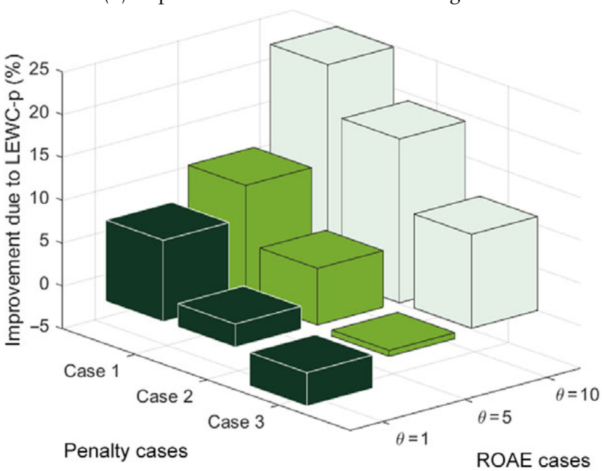
We next test the sensitivity of the gained benefits to the penalty costs and costs associated with adverse events. As we increase the latter, the improvement in the two-hour boarding rate and the average ED boarding time increases (see Figures 18.8(a) and 18.8(b)). Furthermore, we observe that as we increase the penalty cost, IWs start to work as dedicated units tending to only serve their primary patients. Hence, after increasing the penalty cost, we observe improvements in overflow proportions, but the average ED boarding time and the costs associated with adverse events increase. This result is similar to what we observed from the optimal policy of the analytical model: As we increase (decrease) the penalty cost, the LEWC-p policy mimics the optimal policy by decreasing (increasing) the assignments to secondary IWs. Similarly, as we increase (decrease) costs associated with adverse events that may occur during ED boarding, the LEWC-p policy mimics the optimal policy by increasing (decreasing) the assignments to secondary IWs.

Figures 18.9(a) and 18.9(b) illustrate the change in the total number of boarded patients in the ED and overflow proportion as ROAE and penalty cost parameters change under the LEWC-p policy. As the ROAE cost increases, the proposed policy starts to assign patients to their secondary IW more aggressively. This leads to a lower average number of patients boarded in the ED and suggests that utilizing a secondary IW is a more attractive option for

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press



(a) Improvement in two-hour boarding rate

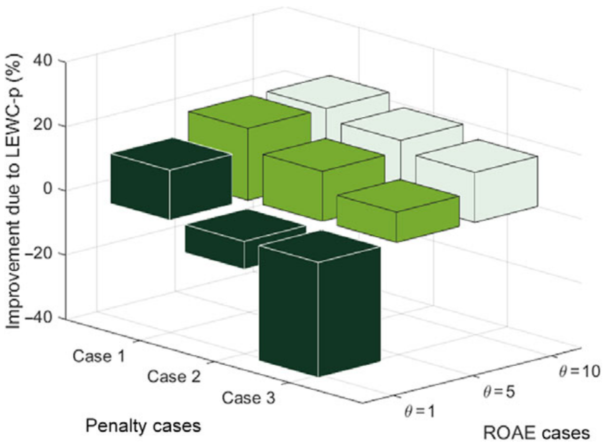


(b) Improvement in average boarding time

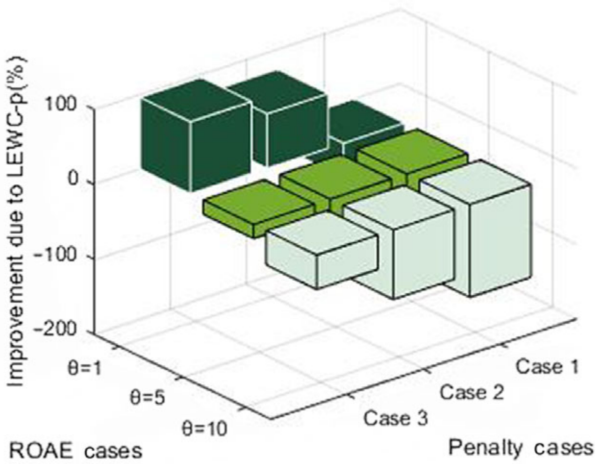
FIGURE 18.8 Improvement due to LEWC-p compared to current practice for various penalty and ROAE parameters

patients who have a higher ROAE (e.g., those in need of timely care following their ED service). Another implication of Figures 18.9(a) and 18.9(b) is that assigning patients to their secondary IWs has a minimal effect on the average ED boarding time when the penalty cost parameter is high. This suggests that hospital administrators should be more patient in assigning beds for patients who are more sensitive to a secondary IW assignment (e.g., type 1 patients as opposed to type 2 patients): *The virtue of patience depends on the patient type.*

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press



(a) Average number of patients boarded in the ED



(b) Overflow proportion

FIGURE 18.9 Effect of ROAE and penalty cost parameters on the average number of patients boarded and overflow proportion due to LEWC-p

18.6.3.1 Effect of Idling Beds

Our proposed policy allows idling IW beds (in anticipation of future needs) even when there are patients boarded in the ED who need them. However, hospital beds are valuable assets, and keeping them idle while patients are waiting for them might not be perceived as attractive by hospital administrators. To gain some insights into the impact of idling, we modify our policy by assigning 4 West patients to 5 West when there is no 5 West patient boarded in the ED (disallowing idling of 5 West beds). From our results on

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

the performance of LEWC-p policy with and without idling, we can make the following observation:

> **Observation 18.2 (Nonidling Policy)** *Nonidling policies increase the number of patients overflowed but do not significantly change the average number of patients boarded in the ED, the average boarding time, or the two-hour boarding rate.*

This observation captures one of the most fundamental trade-offs in our study. Prohibiting idling 5 West beds increases the number of 4 West patients assigned to 5 West while reducing the number of 4 West patients boarded in ED who are eligible for a secondary unit assignment. However, these assignments result in blocking the access of future arriving 5 West patients to 5 West beds, which increases the number of 5 West patients boarded in ED. As a result, the average number of patients boarded in ED, the average boarding time, and the two-hour boarding rate do not change significantly. These outcomes contradict the prevalent perception among hospital administrators that beds should not be idled intentionally. We note that this perception might be correct when the ROAE among different patient groups (in our case, 4 West and 5 West patients) is significantly different.[21] However, our results suggest that hospitals should typically refrain from prohibiting idling: *Idling IW beds can be beneficial.*

18.6.3.2 Effect of Inpatient Bed Capacity

In our previous simulation experiments, we used the current bed capacity of IWs in our partner hospital (see, Table 18.EC.1). To generate insights for other hospitals which might have higher or lower capacities, we now provide sensitivity analysis by altering the number of beds in IWs (both 4 West and 5 West). Figure 18.10 illustrates the results, and enables us to make the following obsevation:

> **Observation 18.3 (Effect of Inpatient Bed Capacity)** *The achievable improvements due to implementing LEWC-p (on the average boarding time, the two-hour boarding rate, and the total cost) is greater in hospitals with lower inpatient bed capacity (all else equal).*

This observation suggests that hospitals that lack enough inpatient bed capacity (e.g., busy teaching hospitals) will benefit more from implementing the LEWC-p policy. Thus, instead of investing in increasing their capacity – a challenging and extraordinarily expensive undertaking that often requires a certification of need – they can benefit from better bed assignment policies such as LEWC-p, which requires only a minimal investment.

[21] If the ROAE for eligible 4 West patients is much larger than that of the 5 West patients, prohibiting idling can be beneficial in terms of the total average cost metric and average boarding time for 4 West patients.

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

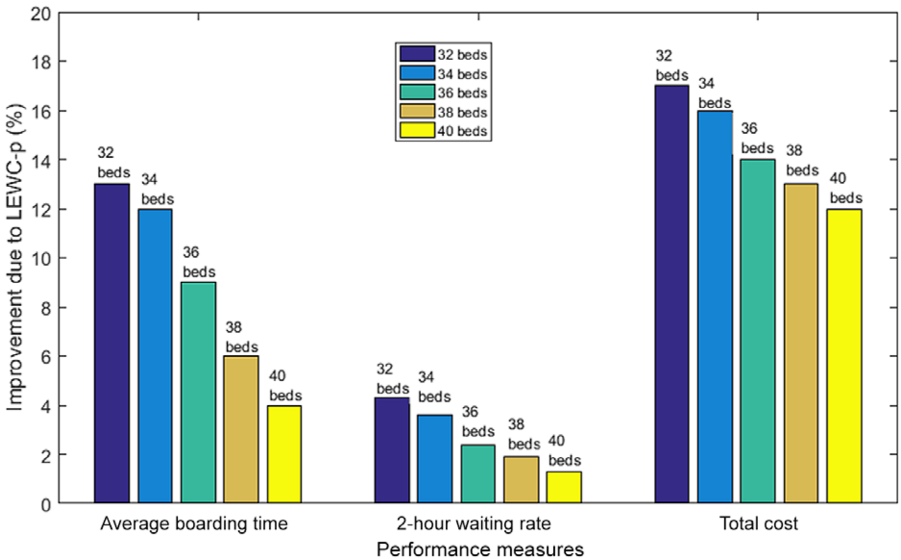


FIGURE 18.10 Effect of inpatient bed capacity on the improvements due to LEWC-p

Another related issue in understanding the effect of inpatient bed capacity is the practice of "bed reservation." Unlike our partner hospital, some hospitals reserve a portion of their IW capacity for their primary patients so as to reduce the effect of overflows on those patients. It is clear that as the number of beds usable for overflows decreases, the number of patients that are assigned to a secondary IW decreases, which in turn results in a lower total penalty cost. However, the impact of this practice on other performance measures such as the average boarding time, the average number of patients boarded, and the two-hour boarding is not obvious. To understand the effect of this practice, we consider three cases by assuming that only 25% (nine beds), 50% (18 beds), and 75% (27 beds) of the beds in 5 West can be used for accommodating CP and CHF patients. From this analysis, we make the following observation:

> **Observation 18.4 (Effect of Restricting Bed Capacity)** *Under the proposed LEWC-p policy, restricting the bed capacity for overflow patients significantly increases the average boarding time, the average number of patients boarded, and the two-hour boarding rate, while decreasing the number of overflows. However, the relative impact of this practice is not statistically different between cases with a 25% and 50% restriction or with a 50% and 75% restriction.*

Our results suggest that, when the number of beds usable for overflows decreases, the number of overflows decreases (as expected). Because our

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

proposed policy captures the trade-off between the ROAE and the quality of care, reduction in overflows leads to an increase in the number of patients boarded in the ED. However, the changes in performance measures are not significant when we either drop to 25% bed capacity from 50% bed capacity or drop to 50% bed capacity from 75% bed capacity. For hospitals with similar characteristics to our partner hospital (in terms of bed request arrivals, inpatient LOS, etc.) this suggests that, to make a statistically significant impact on the performance measures, hospital administrators should consider dramatic changes in the number of beds they use for overflows.

18.6.3.3 Effect of Overflow Trigger Times

Overflow trigger times are often used in practice (see, e.g., Shi et al., 2015) where a patient is overflowed to a secondary IW only when the boarding time of the patient exceeds a predetermined trigger time. We next investigate how our proposed policy performs when the hospital employs an overflow trigger time. To this end, we assume that a patient can be overflowed either when their boarding time exceeds the trigger time or when the LEWC-p policy assigns them to a secondary IW. We analyze the performance of this modified policy by considering various trigger times. From our results, we observe the following:

> **Observation 18.5 (Overflow Trigger Time)** *Imposing overflow trigger times typically increases the penalty costs accrued due to lower levels of quality of care. However, regardless of the level of the trigger time, the relative improvement in the costs associated with adverse events is not high enough to yield an overall improvement in the aggregate cost measure. In addition, the impact of imposing a trigger time on the average boarding time and the two-hour boarding rate is significant only when the trigger time is no more than two hours.*

This observation suggests that imposing an overflow trigger time that is higher than two hours does not change the performance of our proposed LEWC-p policy. In fact, using a trigger time that is more than two hours typically adds complexity in assignment decisions without any significant change in performance measures. As we noted earlier, our proposed LEWC-p policy improves the average boarding time by approximately 10 minutes per patient compared to the current practice. This results in the two-hour boarding rate measure being about 29% in the improved system. Setting a trigger time that is lower than two hours, thus, can affect more than 70% of the patient population (because the two-hour boarding rate measure is about 29%), which may result in improvements in the average boarding time and the two-hour boarding rate. However, we find that adding trigger times to LEWC-p does not lead to improvements in the aggregate cost measure, regardless of the level

https://doi.org/10.1017/9781009483766.019 Published online by Cambridge University Press

of the trigger time.[22] This further confirms that the proposed LEWC-p policy already strikes a strong balance between concerns related to prolonged ED boarding times and those related to overflows.

18.7 CONCLUSION

In this chapter, we shed light on improving the efficiency and the effectiveness of care delivery in hospitals via an innovative way of dynamically assigning ED admitted patients to inpatient units. We make use of a queueing framework and an MDP model to gain insights into effective mechanisms to minimize the risk of adverse events (a patient safety concern) while reducing the number of secondary inpatient unit assignments (a quality of care concern).

Our results for a simplified model with two patient classes and two IWs suggest that the optimal policy is a threshold-type policy, where the threshold depends on the number of patients boarded in the ED. Under this policy, the primary unit of class 1 patients (i.e., patients that have a higher $\theta\mu$ value) typically works as a dedicated unit that serves its primary patients whenever such a patient is boarded in the ED. Moreover, the primary unit of class 2 patients serves them before helping IW 1 on class 1 patients and switches to serving class 1 patients once the number of class 1 patients boarded in the ED reaches a threshold. These suggest that patience in transferring ED admitted patients to IWs is a virtue, but only up to a point. Contrary to the prevalent perception among hospital administrators, we also find that idling IW beds can be beneficial. In particular, although idling is used in some hospitals and some specific inpatient units, our results indicate evidence for wider implementation of idling policies. We also show that, when the penalties that represent the reduction in quality of care in secondary units are negligible, the optimal policy is a strict priority rule in which both IWs prioritize serving class 1 patients in order to myopically decrease the risk of adverse of events for patients boarded in the ED.

Our analyses show that the optimal policy is complex in general and may not be suitable for implementation in practice. Therefore, we make use of the insights gained from analyzing our simplified models to develop two heuristic policies that are easy to implement. We first use a birth-and-death process to approximate the threshold level that minimizes an aggregate measure of both patient safety and quality of care. We then propose a modified version of the LEWC heuristic termed LEWC-p that enables us to dynamically strike a balance between concerns of patient safety and quality of care. The results show that LEWC-p significantly outperforms other policies and is also more robust in that it has a lower standard deviation of optimality gap. Thus, an important

[22] See Appendix 18.G for analysis on performance of pure trigger-based overflow policies as well as related discussions on the differences between LEWC-p and such policies.

contribution of this study is to introduce LEWC-p as a simple but effective policy that can be implemented in hospitals.

We then investigate the achievable gains due to implementing LEWC-p by using a simulation model that we calibrated with a year of data collected from our partner hospital. By using this simulation model, we are able to reflect the realistic features of the hospital patient flow and test the insights gained from our analytical models. To gain clear results, we focus on chest pain (CP) and congestive heart failure (CHF) patients. Furthermore, by utilizing laboratory findings to separate patients based on the level of Tn for CP patients and BNP for CHF patients, we classify these patients as type 1 and type 2. Our analyses of CP and CHF patients indicate that LEWC-p can yield significant improvements compared to the current practice by striking a better balance between patient safety and quality of care metrics. We also shed light on various hospital characteristics that can make the use of our proposed policy more beneficial.

We suspect that the proposed model and policy on patient flow from the ED to IWs can be extended to other areas of the hospital. Similar to the bed-block phenomenon in the ED, operating rooms (ORs) experience problems due to bed shortages in the postanesthesia care unit (PACU). Our model and analyses can be used in those areas of a hospital to provide insights into the trade-off between waiting to be assigned to an appropriate bed versus a quick overflow to a less appropriate bed.

Our model can be also extended in various other ways. First, our model focuses on the patient flow between the ED to IWs without including the transfer between IWs (the requirements for such a flow would be different from our focus in this chapter). However, an extension of our model can be used to study patient flow between IWs and, hence, may provide other ways to further reduce ED boarding times. Second, our model considers IW beds as servers, although in the actual system the transfer process from ED to IWs is more complicated. For example, in many hospitals, the nurses' availabilities often affect the patient flow, especially when nurses are responsible for transferring patients from the ED to IWs. Future research can expand our study by considering such more complex scenarios. Furthermore, in our objective function, we focus on the risk of adverse events and quality of care of patients admitted through the ED. Future research can extend our objective function by incorporating other concerns such as the LOS and waiting times for ED patients who are discharged home after their ED visit. Using ideas from the theory of two-sided matching (see, e.g., Roth and Sotomayor, 1992) can be another fruitful direction for future research because it can help finding better ways of matching ED patients with IW beds.

Finally, we note that use of technological advancements such as those in telemedicine (see, e.g., Saghafian et al., 2018) or in mobile health (mHealth) (see, e.g., Saghafian and Murphy, 2021) can help hospitals mitigate their overcrowding concerns and, hence, prolonged boarding of their patients. These technological advancements offer new avenues for future researchers to help hospitals improve the efficiency and effectiveness of their care delivery. Policymakers also play an important role in helping hospitals achieve this goal. For example, by making data related to the efficiency and effectiveness of hospitals publicly available through policy levers such as public reporting (see, e.g., Saghafian and Hopp, 2019; Saghafian and Hopp, 2020), or by offering support mechanisms that disincentivize hospitals from cutting value-added steps when faced with demand spikes (see, e.g., Song and Saghafian, 2022; Song et al., 2023; Saghafian et al., 2022c), policymakers can pave the way for hospitals to improve their efficiency and effectiveness. However, achieving large-scale impact requires close corroborations among policymakers, researchers, and hospital administrators, and we hope to see more efforts in this vein in the near future.

ACKNOWLEDGMENTS

This work was supported in part by the Mayo Clinic through grant XSS0133.

REFERENCES

Andradóttir, S., Ayhan, H., and Down D. G. (2007) Compensating for failures with flexible servers. *Operations Research* 55(4):753–768.

Armony, M., and Bambos, N. (2003) Queueing dynamics and maximal throughput scheduling in switched processing systems. *Queueing systems* 44(3):209–252.

Armony, M., and Ward A. R. (2010) Fair dynamic routing in large-scale heterogeneous-server systems. *Operations Research* 58(3):624–637.

Bell, S. L., and Williams, R. J. (2001) Dynamic scheduling of a system with two parallel servers in heavy traffic with resource pooling: asymptotic optimality of a threshold policy. *Annals of Applied Probability* 608–649.

Bernstein, S. L., Aronsky, D., Duseja, R., Epstein, S., Handel D., Hwang, U., McCarthy, M., McConnell, J. K., Pines, J. M., and Rathlev, N., et al. (2009) The effect of emergency department crowding on clinically oriented outcomes. *Academic Emergency Medicine* 16(1):1–10.

Berry Jaeker, J. A., and Tucker, A. L. (2016) Past the point of speeding up: The negative effects of workload saturation on efficiency and patient severity. *Management Science* 63(4):1042–1062.

Best, T. J., Sandıkçı, B., Eisenstein D. D., and Meltzer D. O. (2015) Managing hospital inpatient bed capacity through partitioning care into focused wings. *Manufacturing & Service Operations Management* 17(2):157–176.

Brown L, Gans N, Mandelbaum A, Sakov A, Shen H, Zeltyn S, Zhao L (2005) Statistical analysis of a telephone call center: A queueing-science perspective. *Journal of the American Statistical Association* 100(469):36–50.

Buyukkoc, C., Variaya, P., and Walrand, J. (1985) c mu rule revisited. *Advances in Applied Probability* 17(1):237–238.

Carr, B. G., Hollander, J. E., Baxt, W. G., Datner, E. M., and Pines, J. M. (2010) Trends in boarding of admitted patients in us emergency departments 2003–2005. *Journal of Emergency Medicine* 39(4):506–511.

Chan, C. W., Farias, V.F., and Escobar, G. J. (2016) The impact of delays on service times in the intensive care unit. *Management Science* 63(7):2049–2072.

Clark, J. R., and Huckman R. S. (2012) Broadening focus: Spillovers, complementarities, and specialization in the hospital industry. *Management Science* 58(4):708–722.

CNN (2008) Tape shows woman dying on waiting room floor. www.cnn.com/2008/US/07/01/waiting.room.death.

Cox, D.R., and Smith, W. (1991) *Queues*, volume 2 (CRC Press).

Dai, J. G., and Lin, W. (2005) Maximum pressure policies in stochastic processing networks. *Operations Research* 53(2):197–218.

Dai, J., and Shi, P. (2019) Inpatient overflow: An approximate dynamic programming approach. *Manufacturing & Service Operations Management*.

De Véricourt, F., and Zhou, Y. P. (2005) Managing response time in a call-routing problem with service failure. *Operations Research* 53(6):968–981.

Dong, J., Shi, P., Zheng, F., and Jin, X. (2018) Capacity management in inpatient wards with off-service placement and a network view. Available at SSRN 3306853.

GAO (2003) Hospital emergency departments: Crowding vary among hospitals and communities. www.gao.gov/new.items/d03460.pdf.

(2009) Hospital emergency departments: Crowding continues to occur, and some patients wait longer than recommended time frames. www.gao.gov/new.items/d09347.pdf.

Garnett, O., and Mandelbaum, A. (2000) An introduction to skills-based routing and its operational complexities. *Teaching notes* 114.

Gesensway, D. (2010) Having problems finding your patients? www.todayshospitalist.com/Having-problems-finding-your-patients/.

Griffin, J. A. (2012) *Improving health care delivery through multi-objective resource allocation*. Ph.D. thesis, Georgia Institute of Technology.

Gurvich, I., and Perry, O. (2012) Overflow networks: Approximations and implications to call center outsourcing. *Operations research* 60(4):996–1009.

Gurvich, I., and Whitt, W. (2009) Scheduling flexible servers with convex delay costs in many-server service systems. *Manufacturing & Service Operations Management* 11(2):237–253.

Harrison, G. W., Shafer, A., and Mackay, M. (2005) Modelling variability in hospital bed occupancy. *Health Care Management Science* 8(4):325–334.

Hodgson, N., Saghafian, S., Mi, L., Buras, M., Katz E., Pines, J., Sanchez, L., Silvers, S., Maher, S., and Traub, S. (2018) Are testers also admitters? Comparing emergency physician resource utilization and admitting practices. *American Journal of Emergency Medicine* 36(10):1865–1869.

Hoot, N. R., and Aronsky, D. (2008) Systematic review of emergency department crowding: Causes, effects, and solutions. *Annals of Emergency Medicine* 52(2): 126–136.

Kakalik, J. S., and Little, J. D. (1971) Optimal service policy for the m/g/1 queue with multiple classes of arrivals. Technical report, Rand Corporation, Santa Monica, CA.

Kc, D. S., and Terwiesch, C. (2011) The effects of focus on performance: Evidence from california hospitals. *Management Science* 57(11):1897–1912.

Kuntz, L., Mennicken, R., and Scholtes, S. (2014) Stress on the ward: Evidence of safety tipping points in hospitals. *Management Science* 61(4):754–771.

Lin, W., and Kumar, P. (1984) Optimal control of a queueing system with two heterogeneous servers. *IEEE Transactions on Automatic Control* 29(8):696–703.

Mandelbaum, A., Momčilović, P., and Tseytlin, Y. (2012) On fair routing from emergency departments to hospital wards: QED queues with heterogeneous servers. *Management Science* 58(7):1273–1291.

Mandelbaum, A., and Stolyar, A. L. (2004) Scheduling flexible servers with convex delay costs: Heavy-traffic optimality of the generalized cμ-rule. *Operations Research* 52(6):836–855.

Meyn, S. P. (2003) Sequencing and routing in multiclass queueing networks part II: Workload relaxations. *SIAM Journal on Control and Optimization* 42(1):178–217.

Powell, E. S., Khare, R. K., Venkatesh, A. K., Van Roo, B. D., Adams, J. G., and Reinhardt, G. (2012) The relationship between inpatient discharge timing and emergency department boarding. *Journal of Emergency Medicine* 42(2):186–196.

Proudlove, N., Boaden, R., and Jorgensen, J. (2007) Developing bed managers: The why and the how. *Journal of Nursing Management* 15(1):34–42.

Rabin, E., Kocher, K., McClelland, M., Pines, J., Hwang, U., Rathlev N., Asplin, B., Trueger, N. S., and Weber, E. (2012) Solutions to emergency department "boarding" and crowding are underused and may need to be legislated. *Health Affairs* 31(8):1757–1766.

Roth, A. E., and Sotomayor, M. (1992) Two-sided matching. *Handbook of Game Theory with Economic Aapplications* 1:485–541.

Saghafian, S., Austin, G., and Traub, S. J. (2015) Operations research/management contributions to emergency department patient flow optimization: Review and research prospects. *IIE Transactions on Healthcare Systems Engineering* 5(2): 101–123.

Saghafian, S., and Hopp, W. (2019) The role of quality transparency in health care: Challenges and potential solutions. *NAM Perspectives. Commentary* (National Academy of Medicine).

(2020) Can public reporting cure healthcare? the role of quality transparency in improving patient-provider alignment. *Operations Research* 68(1):71–92.

Saghafian, S., Hopp, W. Iravani, S., Cheng, Y., and Diermeier, D. (2018) Workload management in telemedical physician triage and other knowledge-based service systems. *Management Science* 64(11):5180–5197.

Saghafian, S., Hopp, W. Van Oyen, M., Desmond, J., and Kronick, S. (2014) Workload management in telemedical physician triage and other knowledge-based service systems. *Manufacturing and Service Operations Management* 16(3):329–345.

Saghafian, S., Hopp, W. J., Van Oyen, M. P., Desmond, J. S., and Kronick, S. L. (2012) Patient streaming as a mechanism for improving responsiveness in emergency departments. *Operations Research* 60(5):1080–1097.

Saghafian, S., Imanirad, R., and Traub, S. (2022a) Do independent and parallel processing physicians influence each other's performance? Evidence from the emergency department. HKS working paper RWP19-026.

(2022b) Who is an effective and efficient physician? evidence from emergency medicine. working paper RWP18-029.

Saghafian, S., and Murphy, S. (2021) Innovative health care delivery: The scientific and regulatory challenges in designing health interventions. *NAM Perspectives. Commentary*. (National Academy of Medicine).

Saghafian, S., Song, L., and Raja, A. (2022c) Towards a more efficient healthcare system: Opportunities and challenges caused by hospital closures amid the COVID-19 pandemic. *Health Care Management Science* 25:187–190.

Saghafian, S., Van Oyen, M. P., and Kolfal, B. (2011) The "W" network and the dynamic control of unreliable flexible servers. *IIE Transactions* 43(12):893–907.

Saghafian, S., and Veatch, M. H. (2016) A c-mu rule for two-tiered parallel servers. *IEEE Transactions on Automatic Control* 61(4):1046–1050.

Sennot, L. (1999) *Stochastic Dynamic Programming and Control of Queueing Systems* (John Wiley & Sons).

Shi, P., Chou, M. C., Dai, J., Ding, D., and Sim, J. (2015) Models and insights for hospital inpatient operations: Time-dependent ed boarding time. *Management Science* 62(1):1–28.

Singer, A. J., Thode Jr., H. C., Viccellio, P., and Pines, J. M. (2011) The association between length of emergency department boarding and mortality. *Academic Emergency Medicine* 18(12):1324–1329.

Song, H., Tucker, A., Graue, R., Moravick, S., and Yang, J. (2019) Capacity pooling in hospitals: The hidden consequences of off-service placement. Available at SSRN 3186726.

Song, L., and Saghafian, S. (2022) The spillover effects of hospital closures on the efficiency and quality of other hospitals. HKS working paper RWP19-006.

Song, L., Saghafian, S., Newhouse, J., Landrum, M., and Hsu, J. (2023) The impact of vertical integration on physician behavior and healthcare delivery: Evidence from gastroenterology practices. NBER working paper 30928.

Sri-On, J., Chang, Y., Curley, D. P., Camargo Jr., C. A., Weissman, J. S., Singer, S. J., and Liu, S. W. (2014) Boarding is associated with higher rates of medication delays and adverse events but fewer laboratory-related delays. *The American Journal of Emergency Medicine* 32(9):1033–1036.

Teow, K. L., El-Darzi, E., Foo, C., Jin, X., and Sim, J. (2012) Intelligent analysis of acute bed overflow in a tertiary hospital in singapore. *Journal of Medical Systems* 36(3):1873–1882.

Thompson, S., Nunez, M., Garfinkel, R., and Dean, M. D. (2009) Or practice – efficient short-term allocation and reallocation of patients to floors of a hospital during demand surges. *Operations Research* 57(2):261–273.

Traub, S., Bartley, A., Didehban, R., Smith, V., Lipinski, C., and Saghafian, S. (2016a) Physician in triage versus rotational patient assignment. *ournal of Emergency Medicine* 50(5):784–790.

Traub, S., Saghafian, S., Judson, K., Russi, C., Madsen, B., Cha, S., Tolson, H., Sanchez, L., and Pines, J. (2018) Interphysician differences in emergency department length of stay. *Journal of Emergency Medicine* 54(5):702–710.

Traub, S., Stewart, C., Didehban, R., Bartley, A., Saghafian, S., Smith, V., Silvers, S., LeCheminant, R., and Lipinski, C. (2016b) Emergency department rotational patient assignment. *Annals of Emergency Medicine* 67(2):206–215.

Traub, S., Stewart, C., Didehban, R., Nestler, D., Chang, Y., Saghafian, S., and Lipinski, C. (2015) Emergency department rapid medical assessment: Overall effect and mechanistic consideration. *Journal of Emergency Medicine* 48(5):620–627.

Traub, S., Temkit, M., and Saghafian, S. (2017) Emergency department holding orders. *Journal of Emergency Medicine* 52(6):885–893.
Van Mieghem, J. A. (1995) Dynamic scheduling with convex delay costs: The generalized $c\mu$ rule. *The Annals of Applied Probability* 809–833.
Wallace, R.B., and Whitt, W. (2005) A staffing algorithm for call centers with skill-based routing. *Manufacturing and Service Operations Management* 7(4):276–294.
Welch, P. D. (1983) The statistical analysis of simulation results, in the computer performance modeling handbook. *The Computer Performance Modeling Handbook* 22:268–328.
Zhan, D., and Ward, A. R. (2013) Threshold routing to trade off waiting and call resolution in call centers. *Manufacturing and Service Operations Management* 16(2):220–237.